Innovative
Policy Modelling and Governance Tools
for Sustainable Post-Crisis Urban Development

D2.3 Review of Urban Models: Use in Urban Policy

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Executive Summary

In this work we present a classification of urban modelling paradigms. The classification is intended to help us frame the modelling paradigms within the objectives of the INSIGHT project. Seven main different categories have been identified: optimisation location-allocation models; top-down static models; top-down dynamic spatially explicit models; top-down dynamic non-spatially explicit models; bottom-up static models; bottom-up dynamic spatially explicit models; and bottom-up dynamic non-spatially explicit models. We discuss in detail the characteristics of these categories and describe the different modelling paradigms.

We also present a brief discussion of the advantages and disadvantages of the different models in the context of their use for policy making. We conclude that no single modelling paradigm is sufficient to give an answer for the whole diversity of areas affected by urban development and meet the needs and requirements of all the actors involved in the policy making process, but rather the combination of different models in a complementary fashion is needed.

Finally we review a list of state-of-the-art tools used in the context of urban planning. The tools which are candidates to be used in the project are reviewed in more detail, and other tools are briefly discussed.

1. Introduction

1.1 Scope and objectives

In this work we present a review of different urban simulation models and decision support tools existing both in academic and in operational contexts, including LUTI models as well as more specific location models for housing, retail, and public services.

The objective is to produce a classification of urban modelling paradigms aligned with the INSIGHT objectives. Such classification will help categorise different state-of-the-art urban planning decision support tools and evaluate their relevance for project.

1.2 Methodology

The followed methodology consists mainly in a literature review, complemented by the results of the stakeholder consultation on urban models and decision support tools performed within the framework of the project and documented in deliverable D2.1 Stakeholder Consultation Report.

1.3 Description of contents

In section 2 we discuss the relevance of urban models and decision support tools for the process of urban planning and policy making. A modelling paradigm classification and a detailed description of the models belonging to each of the identified categories are presented in section 3. Section 4 presents a brief discussion of the weaknesses and strengths of the different models as decision support tools for policy making. Section 5 includes a review of different state-of-the-art land use tools, describing their main features and technical characteristics.

2. Urban modelling and decision support tools for planning and policy making

Urban planning concerns collective decision making aimed at the design and implementation of courses of action in order to achieve future societal goals to resolve collective problems that actors in a reference system experience and define (Alexander, 1992; Faludi, 2000; Huxley and Yiftachel, 2000). It encompasses research and analysis, strategic thinking, architecture, urban design, public consultation, policy recommendations, implementation and management (Taylor, 2007). Land use policy and planning are not limited to direct physical interventions; they are often exercised indirectly through actions in economic, social, environmental, and other policy and planning arenas (Healey, 2003). Conversely, land use policies and plans serve as instruments to promote economic, social, environmental, and other goals (Briassoulis, 2007).

To deal with the complexity involved with planning and policy making, urban planners need models and tools assisting them in process of policy evaluation and results optimisation. A *model* is to be understood as a simplified description of a complex entity or process, often in mathematical terms, that helps conceptualise and analyse the problem. A *tool* is to be understood in this context as a specific software kit or computer program. In this document we focus on those tools implementing specific versions of the models previously described.

Urban models following different approaches have been developed, going from the earlier top-down models to the recent bottom-up and participatory models. Waddell and Ulfarson (2003) defend the importance of including participatory practices during the policy making process, as well as the explicit inclusion in urban models of socio-economic, environmental and other aspects affected by a policy implementation. According to Waddell and Ulfarson (2003) a useful urban model for policy making should have the following characteristics: (i) be sensitive to a range of land use and transportation policies and their interactions; (ii) build on clear and defensible behavioural foundations; and (iii) facilitate participation in the testing of alternative policy strategies and their evaluation. Advances such as discrete choice theory and agent-based modelling have been allowed advancing in these directions, by further disaggregating zonal components and also population and employment groups using socio-economic attributes (Serras et al., 2014).

In parallel to the development of new modelling paradigms, progress in ICT have contributed to the development of more realistic and user friendly urban modelling tools. Geographic Information Systems (GIS) have facilitated the availability of georeferenced data and provided user-friendly tools for the visualisation of models results. Other ICT have begun to offer tools for public engagement through new forms of data representation (Hanzl, 2007), as well as new ICT-enabled social participation techniques, such as charrettes, brainstorming and buzz sessions, synectic sessions, or take-part workshops supported by Planning Support Systems, Participatory Planning GIS, simulation and role-paying games (Innes and Booher, 2000; Sanoff, 2000; Condon et al 2009; Steiniger et al 2012).

Different urban modelling paradigms and tools and their relevance to policy assessment are discussed in the following sections.

3. Model classification

There are many ways of characterising urban models, and there are few models that fully fit into one distinct category. Different classifications have been proposed in the literature (see e.g. Anderstig and Mattsson, 1998; Batty, 2009; Birkin and Heppenstall, 2011; Briassoulis, 2007; Couclelis, 2002; DSC and MEP, 1999; Hamacher and Nickel, 1998; Wegener, 2004). Models can be classified by their purpose, structure, analytical approach (top-down vs bottom-up), underlying methodology, mathematical approach (optimisation vs simulation), sectorial coverage, time horizon, or data requirements, among other criteria. This review does not claim to introduce new concepts or develop original theory but to propose a model classification allowing the evaluation of the relevance of different models and tools to the INSIGHT project.

3.1 Classification criteria

The selected criteria for a classification of different land-use modelling are the following:

1. Is the influence of different elements of the system (users, services, housing, etc.) captured by the model?
    - Are relationships between different elements of the system considered?
    - If so, are they considered as exogenous/endogenous parameters?
2. Are these models able to capture emergent properties?
    - Bottom-up Vs Top-down
3. Does the model consider time evolution?
    - Static Vs Dynamic
4. Is spatial information explicitly considered?

3.2 Model tree

Based on the previous classification criteria, we build a model tree where seven main categories have been identified (see Figure 1):

- Optimisation (prescriptive) location-allocation models
- Top-down static models
- Top-down, dynamic, spatially explicit models
- Top-down, dynamic, non-spatially explicit models
- Bottom-up static models
- Bottom-up, dynamic, spatially explicit models
- Bottom-up, dynamic, non-spatially explicit models

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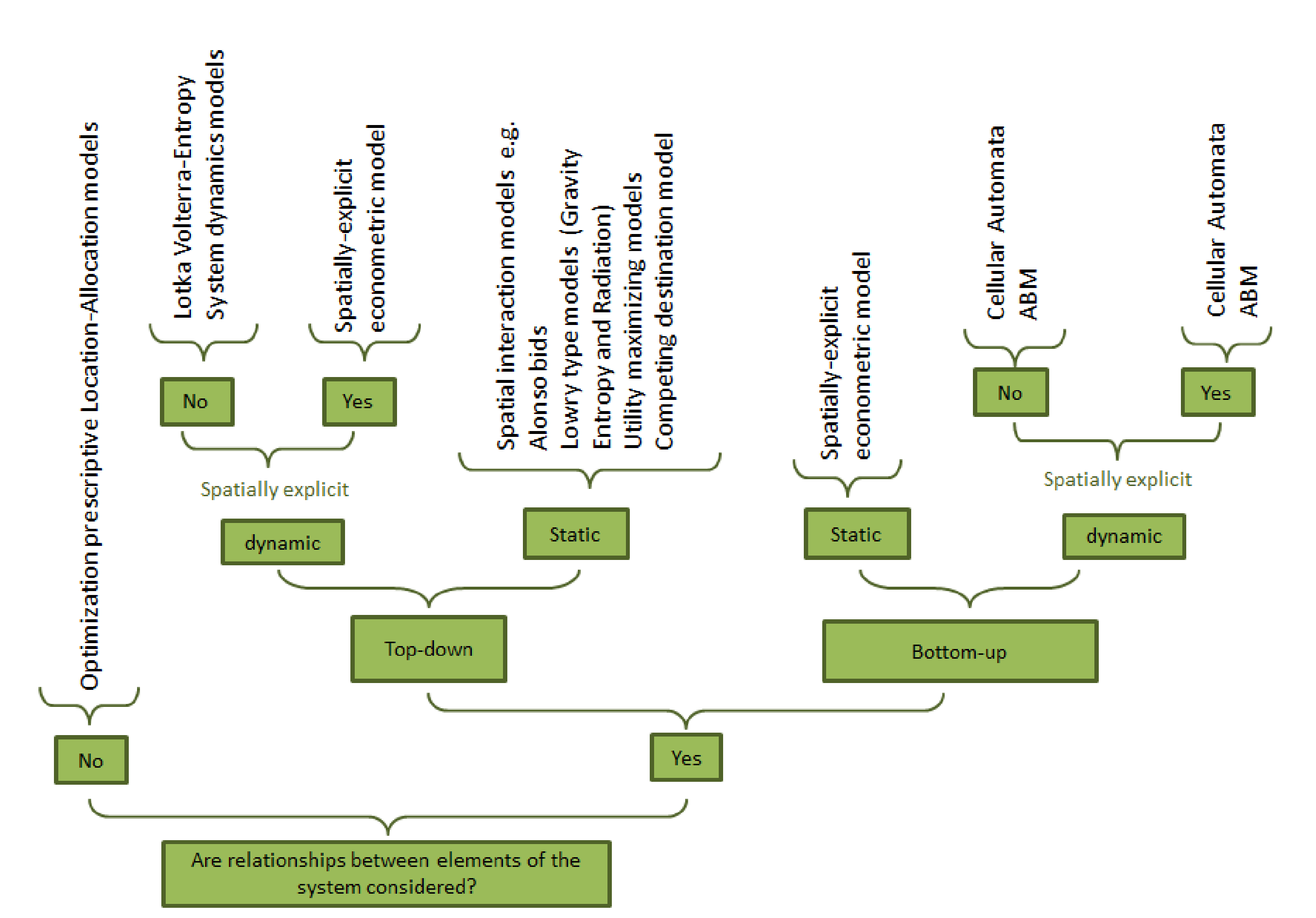

Figure 1. Model classification tree

"Is the influence of different elements of the system (users, services, housing, etc.) captured by the model?" Does the model consider the interaction between different elements of the system (in terms of consideration of ''the interactions, relationships, and linkages between two or more components of a spatial system'')?

A first separate branch of models corresponds to **optimisation (prescriptive) location- allocation models**. Location-allocation models work as "how to" tools rather than as "what if" tools. Following maximisation principles they identify the best solution among different known possible options. In the case of urban planning they are used to find the most appropriate location for a given service. Depending on the nature of the sector (retail, public services, private services, etc.), different problems arise, including the p-median problem, the location set covering problem, the maximal covering location problem, and their different extensions.

All other models considered in this review can be considered as **interaction models**. This interaction may be between different elements of the same sector or activity, such as the interactions between two different kinds of land use (housing and retail for instance), or between different activities or elements of the system, being Land Use Transport Integrated (LUTI) models the most common. LUTI models consider the interaction between population, land use and transport services (for a detailed review, see e.g. Wegener, 2004).

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Interaction models may adopt different approaches and consider interaction at different levels. To go further in model classification, we now consider the approach taken to model the relation between elements.

"Is the model able to capture emergent properties?" Bottom-up Vs Top-down

Top-down models predict future or explain behaviours based on tested behavioural hypothesis using statistics on aggregated historical data. In these models related state variables are aggregated, e.g. through differential equations. Top-down urban models can be sub-classified into dynamic and static models depending on whether or not they consider the time evolution of the system.

- **Top-down static models**. In this branch we can find the Lowry type spatial interaction models. These models consider one of the fundamental relationships in urban planning: that of transport and land use. They specify the interaction between transportation networks and location as a set of aggregated relationships based on the behaviour of a representative individual, usually the mean calculated from a representative sample of the population. Their main concern is to describe/predict flows between two different zones. The temporal dependence is averaged out and so flows are considered as time independent. The relevant variables of these models can be population density, travel cost, job offer, etc.
- **Top-down dynamic models**. The current state of the system is considered as a consequence of the interaction between variables and states of the different elements of the system at a previous time step. The "initial states" of the different elements and the interaction between them are averaged quantities usually obtained empirically from historical data. The equilibrium state may or may not be reached. Spatial characteristics are not necessarily considered as endogenous parameters of the system.
    - **Top-down dynamic non-spatially explicit models**. Here we find models developed under a system dynamics approach and also hybrids of static spatial interacting models and dynamical models inherited from biology, such as Lotka-Volterra models. These hybrids models consider fast dynamics to be in equilibrium (i.e., equilibrium is assumed at each time step and daily flows are calculated using e.g. gravity type models), while the slow dynamics (i.e., the long term structural changes of the system) is modelled using predator-prey type models.
    - **Top-down dynamic spatially explicit models**. Here we can find spatially-explicit econometric models. These models, built on a system dynamics approach, estimate economic parameters through specific space characteristics such as distance to city centre, accessibility to zones, etc.

Moving to a different branch of the classification tree we find **bottom-up models**. In bottom-up models the observed dynamics of the system is the result of the individual behaviour of agents or actors (individuals or groups) and the interaction between them. These models are usually focused on decisions rules in which agents behave in response to their environment and to each other. They are able to simulate emergent phenomena and are capable of embodying novelty, surprise and innovation.

- **Bottom-up dynamic models.** By definition all bottom-up models are dynamic, since they consider the time evolution of the system due to the interaction between agents. However such systems may reach a temporal static or dynamic equilibrium and the observed emergent phenomena may correspond to an equilibrium state. **Agent-based models** and **cellular automata models** correspond to this model category. These models may or may not consider spatial properties as endogenous variables of the system.

- **Bottom-up "static" models** are a special case of CA or ABM where decision rules are used to allocate land use changes. These decision rules describe the relationships between land use and human and biophysical factors and depend on some observed probability function. These models are considered static in the sense that the probability function is not updated as the neighbouring cells change. The system eventually reaches an equilibrium state and becomes "static". Spatially-explicit econometric models fall within this category.

The following sections give a detailed description of the different models within each category.

3.3 Model description

3.3.1 Optimisation prescriptive location- allocation models

Optimisation prescriptive location- allocation models work as a "how to" tool, i.e. they identify the best solution among different known possible options. These models act on maximisation principles, giving the best option in relation to a predefined goal. The goal is represented by a function that has to be maximised according to a series of alternatives and imposed constraints.

In the case of land-use they are used to choose the best location of a facility, for a given land and population distribution. They do not predict what the results or the consequences or impact of placing a facility in a given place will have on other existing facilities or in the demand for that or other service.

Facility location models are concerned with the provision of a service to satisfy a spatially dispersed demand. When a demand for the service exists at a large number of widely dispersed sites, it is impossible to provide the service at every single point and then the best configuration to accomplish with a predefined goal should be found. The goal to be achieved depends on the nature of the facility to be allocated (service provision or retail centre allocation) and on the characteristics and interest of the service provider (public or private provider), e.g. the objective could be to minimise distance/costs or maximise benefits.

For instance, it may make more sense to a city fire department to locate fire stations so that a response to every property is possible in less than five minutes, than to worry about minimising the average response time (Longley et al., 2005), while for a delivery company it may make more sense to locate delivery branches in such a way as to minimise the average distance/cost to all possible demand locations and hence increasing the benefits than locating a larger amount of branches such that every demand location is reached in given minimum time. The goal will also be different if there is a constraint in the number of available facilities or not or in the minimum travel distance.

Different models have been proposed to cover all possible options. The P-Median Problem (PMP), the P-Centre Problem (PCP) and the Location Set Covering Problem (LSCP) give an answer to mandatory covering problems, where the goal is to cover 100% of the demand locations. The Maximal Covering Location Problem (MCLP) and its modifications give an answer to maximal covering problems, where the goal is to maximise the demand locations covered when it is not possible to offer a full coverage.

3.3.1.1 Mandatory covering problems

P-median problem

The **p-median problem (PMP)** first formulated by Hakimi (1964) is one of the basic models in discrete location theory. The objective of the PMP is to find the best location of *p* facilities such that it minimises the total or average distance between them and demands assigned to them, ensuring a full coverage. Distances are measures of impedance; they can be time distances or monetary costs, depending on the problem.

From all possible configurations that allocate exactly p facilities, an optimal solution is chosen. The optimal solution is that which corresponds to the configuration that has the minimal cost. Every p-median problem has at least one optimal solution but there can be more. The mathematical expression of the p-median problem is:

Minimise:

$$z = \sum_i \sum_j a_i d_{ij} x_{ij} \qquad (1)$$

subject to:

$$\sum_j x_{ij} = 1 \text{ and} \sum_j x_{jj} = p \qquad (2)$$

with

$$x_{ij} \le x_{jj,}\ x_{ij} = \begin{cases} 1 \text{ if demand at i is attended by a facility at j} \\ 0 \text{ otherwise} \end{cases}, \quad (3)$$

$$i, j = 1 \dots n \text{ and } p \le n$$

where *z* is the value of the objective function to be minimised, *n* is the number of demand locations, *p* is the number of facilities to be located, d_{ij} is the shortest distance/travel cost from demand location *i* to facility location *j*, a_i is the demand at *i*, and x_{ij} is the decision or spatial allocation variable. The constraint in Eq. 2 ensures that every demand point is attended by one facility and that all *p* facilities are located. The set of the demand points is the same as the candidate of facility location points, therefore both *i* and *j* have the same range from 1 to *n*. An optimisation algorithm is used to find a solution such that the objective function returns a minimal (or maximal) value. Heuristic (Greedy, Drop, Teitz-Bart) randomised (e.g. through Genetic Algorithms and Simulated Annealing) algorithms are usually used. The result of the optimisation process is a configuration with *p* facilities and the total cost calculated by the objective function for this configuration.

The solution of this problem can be of interest where the cost/travel distance minimisation for individual trips is not relevant but the averaged one is, e.g. for retail or delivery companies that want to cover a certain number of neighbourhoods with a given number of branches. However, this problem is not useful when the distance from the facility to the user cannot exceed a certain threshold, such as in emergency services location, e.g. an ambulance service that has to cover a given area within a maximum distance/time in order for the service to be of any use. Also, the P-Median problem (PMP) considers uncapacitated service offer, i.e. it does not take into account the possibility of saturation of a service provider. To address these problems, constraints on maximum distance and/or maximum capacity of service provision can be added to the p-median problem.

P-centre problem

The **p-centre problem (PCP)** is a special case of the PMP where the objective is to minimise the farthest distance (Klose and Drexl, 2005; Hamacher and Nickel, 1998) from the demand points to the facilities rather than the average distance. This may be desired for emergency services, where the response time for the place located furthest away must still be reasonably short. It can also be of interest for services provision allocation when the availability of resources is limited. In the case of education centres, for instance, it ensures that education centres are located such that all neighbourhoods are provided with one in the minimum possible distance. Several modifications of the p-centre problem considering limited coverage have been proposed. Daskin and Owen (1999) introduced the problem of covering a given fraction of the demand with a maximum distance to the facility and with a minimum number of facilities, as well as that of covering a given fraction in such a way that the distance to the closest facility is minimised.

Location set covering problem

In some cases budget and/or space is limited and then the number of facilities has to be reduced by increasing the distance between the facilities and the attended areas. In these cases the optimal solutions to the p-centre or p-median problem are not a good enough or feasible solution and it is necessary to optimise the number of facilities providing a service. In some cases it is preferable to increase the distance between facility and attended area while ensuring that all areas are covered within a standard distance, e.g. fire alarms do not need to be at the minimum distance from every user as long as they are within a reasonable distance to be heard.

To give an answer to these situations, the **location set covering problem (LSCP)** was proposed. The LSCP seeks the minimum number of facilities such that all demands are covered within a standard distance (Toregas and ReVelle, 1972). The mathematical expression of the LSCP is the following:

Minimise:

$$z = \sum_{j \in J} x_j \qquad (4)$$

subject to:

$$\sum_{j \in N_i} x_j \geq 1 \ \forall i \in I \qquad (5)$$

and

$$x_j = \begin{cases} 1 & \text{if a facility is located at site j} \\ 0 & \text{otherwise} \end{cases}, \quad \forall\, j \in J \qquad (6)$$

where *i* is the set of demand nodes, *j* the set of candidate facility locations, and $N_i = \{j | d_{ij} \leq D_i\}$ the set of all candidate locations that can cover demand point *i*, with D_i being the coverage distance and d_{ij} the distance between demand at node *i* and candidate site *j.*

The objective function *z* minimises the number of facilities located while the constraint (5) ensures that each demand node *i* is covered by at least one facility within a distance D_i.

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An underlying assumption of the LSCP is that all the demand nodes must be covered, i.e. there is not a limited number of facilities to be allocated. However in many situations that is not the case and the number of facilities is limited. The LSCP ensures then a complete coverage with the minimum number of facilities but it cannot be considered when the number of facilities is fixed.

3.3.1.2 Maximal covering problems

Complete coverage of all demand in a region is not always possible due to budgetary limitations and so decision makers should opt for a satisfactory solution which complies with a minimum standard. Siting studies where this planning dilemma occurs include placement of emergency warning sirens (Current and O'Kelly, 1992), bus stops (Gleason, 1975), cellular towers (Kalvenes et al., 2004), emergency response stations (Adenso-Diaz and Rodríguez, 1997; Church and ReVelle, 1974; Toregas et al 1971; Marianov and ReVelle, 1996; Gendreau et al., 1997), automatic meter reading stations (Gavirneni, 2004), and radio receiving stations and other types of signal transmitting equipment (Brady and Rosenthal, 1980; Mehrez and Stulman, 1984).

In the case of planning situations which have an upper limit on the number of facilities to be sited, the **maximal covering location problem (MCLP)** proposed by Church and Revelle (1974) can be considered. The objective of the MCLP is to locate a predetermined number of facilities *p* in such a way as to maximise the demand that is covered. MCLP assumes that there may not be enough facilities to cover all of the demand nodes. If not all nodes can be covered, the model seeks the siting scheme that covers most demand. In the MCLP, the number of facilities is therefore known a priori and the objective becomes to maximise the amount of demand covered within the acceptable service distance *D*. Each demand area can be covered not only by one facility but also by two or more so that each facility covers a percentage of demand. The mathematical expression for the MCLP problem is:

Maximise:

$$z = \sum_i w_i x_i \qquad (7)$$

subject to:

$$\sum_j y_{j \in N_i} \geq x_i \quad \forall\, i \in I, \quad (8)$$

$$\sum_i y_i = p \qquad (9)$$

with:

$$x_i = \begin{cases} 1 \text{ if demand node i is covered by at least one facility} \\ 0 \text{ otherwise} \end{cases}, \forall\, i \in I \qquad (10)$$

$$y_j = \begin{cases} 1 \text{ a facility is located at zone j} \\ 0 \text{ otherwise} \end{cases}, \quad \forall\, j \in J \qquad (11)$$

where z is the objective function to be maximised, w_i is the population to be served at node i, $N_i = \{ j \in J |\, d_{ij} \leq D_i \}$ is the set of nodes j with in a maximum service distance D_i for every demand zone i, d_{ij} is the shortest distance/travel cost from demand location i to facility location j, x_i is the server spatial allocation variable, I the set of demand points, J set of all candidate locations and p number of facilities to be allocated.

The MCLP is NP-hard (Church and ReVelle, 1976; Garey and Johnson, 1979). The various solution approaches that have been proposed in the literature over the years include greedy heuristics (Daskin, 1995), Lagrangean relaxation (Galvão and ReVelle, 1996), Lagrangean/surrogate heuristics (Lorena and Pereira, 2002). Detailed reviews of solution approaches to the MCLP can be found in Serra and Marianov (2004) and Galvão (2004).

3.3.1.3 Dynamic location allocation models

The models discussed so far do not explicitly address time dependent problems such as consideration of future scenarios and/or time dependent demand problems. There exist some extensions of them considering time dependent scenarios known as dynamic location-allocation models. According to Current et al. (1998) dynamic location-allocation models can be divided into two categories: "implicitly" dynamic and "explicitly" dynamic.

Implicitly dynamic models are "static" in the sense that all the facilities are to be opened at one time and remain open over the planning horizon. They are dynamic because they recognise that problem parameters (e.g., demand) may vary over time and account for these changes. The problems studied in this category include facilities relocation, where the best times to relocate the facilities and the best location after each relocation is found (Drezner and Wesolowsky, 1991). These models are convenient when the cost of relocation is not high. Relocation can occur in a periodic fashion along one day, week, etc. This is known as redeployment and occurs e.g. in the case of emergency medical services such as ambulances and also car services under demand or delivery services. Gendreau et al (2001) proposed a dynamic double standard model at time t (DDSMt) for redeployment of ambulances, whose objective is to maximise backup coverage while minimising relocation costs. While the primary objective is to maximise the proportion of calls covered by at least two vehicles within a distance threshold, the model penalises repeated relocation of the same vehicle, long round trips, and long trips. The model's input parameters are updated each time a call is received and DDSMt is solved. To solve this complex model, particularly at short time intervals, Gendreau et al. developed a fast tabu search heuristic. Using real data from the Island of Montreal, their tests showed that the algorithm was able to generate new redeployment strategies for 95% of all cases. More work on redeployment of ambulances has been done by Brotcorne et al. (2003), among others. Other examples of implicitly dynamic models are Mirchandani and Odoni (1979) and Weaver and Church (1983).

Explicitly dynamic models are designed for problems where the facilities will be opened (and possibly closed) over time. Typically, explicitly dynamic models extend the basic, static models with the addition of temporal subscripts to the facility location and assignment variables and constraints linking these variables over time. The decision to open and close facilities over time is related to changes in the problem parameters. Examples of parameters that might change include demand, travel time/cost, facility availability, fixed and variable costs, profit, and the number of facilities to be opened. Models in this category are those of location of one or several facilities so as to optimise results over a large period of time. These models are relevant when the variation of demand occurs at longer periods of time and the cost of relocation is high. Drezner and Wesolowsky (1991) consider locating a facility in a growing city with predictable population shifts (i.e., demands change over time in a deterministic manner). Its objective is to find a single facility location which minimises the expected cost over the given horizon. For several facilities location a common approach is the continuous location and relocation of facilities at different time steps, known as multi-objective or multi-period approach. Wesolowsky and Truscott

(1976) proposed a multi-period node location-allocation problem, allowing facilities to be relocated in response to predicted changes in demand. An integer programming model is presented, with a constraint restricting the number of location changes in each period. A dynamic programming formulation is also presented. Drezner (1995) proposed an extension of the p-median problem to account for time dependent demand, the **progressive p-median problem**, where demand is considered time dependent with a given functional relationship; the objective is the successive location of *p* facilities without relocation over a planning horizon of T periods, each with a set of different demands, in such a way as to minimise total transport cost (or distance) over the horizon. Inputs to the progressive p-median problem include time-dependent (known) demands and times at which the facilities are to be located.

An extension of the MCLP to take into account time variable demand for location of the public sector is proposed by Schilling (1980): multi objective dynamic location (MODL). MODL is basically a composite of T MCLPs which seeks to maximise the amount of demand covered within the acceptable service distance D for each period of time t. The mathematical expression for the MODL problem is:

Maximise:

$$\mathrm{z} = \sum_{\mathrm{i}} \mathrm{w_{it}} \mathrm{x_{it}} \;\; \forall\, t = 1, \dots T \qquad (12)$$

considering the constraints

$$\sum_{\mathrm{j} \in \mathrm{N_{it}}} \mathrm{y_{jt}} \geq \mathrm{x_{jt}} \;\; \forall\, \mathrm{i} \in \mathrm{I} \text{ and } \forall\, \mathrm{t} = 1, \dots \mathrm{T}\,, \qquad (13)$$

$$\sum_{\mathrm{j} \in \mathrm{J}} \mathrm{y_{jt}} \geq \mathrm{p_t} \;\; \forall\, \mathrm{t} = 1, \dots \mathrm{T} \qquad (14)$$

that define coverage for periods from 1 to T, and the constraint

$$y_{jt} \geq y_{jt-1} \;\forall\, j \in J \text{ and } \forall\, t = 2, \dots T \qquad (15)$$

which provides temporal continuity. The problems are connected by constraints that stipulate that once a facility is built, it remains in place through the planning horizon. The objective function, *z*, is actually a vector of T individual period objectives which will not, in general, have a unique optimum. Schilling (1980) discusses multi-objective approaches for generating a set of "efficient" solutions for the decision maker to choose between them. The MODL model can also be formulated to permit relocations if the problem setting requires it (Schilling, 1980). Gunawardane (1982) also considers location problems within the public sector. Specifically, he examines several covering problems in which public facilities are located (and possibly relocated) over a planning horizon. Both the set covering and the maximal covering problem formulations are extended to account for a T period planning horizon. Decreasing weights w_t (indexed over all planning periods t) are used as coefficients to the location variables x_{jt} (indexed over potential facility sites j and planning periods t) in the dynamic set covering objective function to encourage postponing facility locations until they are required. Another multi-objective approach is also examined by Min (1988), who considers expanding and relocating public libraries in the Columbus metropolitan area. The criteria considered in choosing library locations include coverage of population, proximity to each community, proximity to facilities being closed, and accessibility to transportation routes or parking lots. A discrete location model based on fuzzy goal programming is formulated as a mixed integer program. The model is solved multiple times using inputs from the decision maker to obtain a number of potential siting configurations and to illustrate trade-offs between objectives.

Stochastic location problems

The previously discussed dynamic models assume that the parameters of the problem are known with certainty. However, there is considerable uncertainty in most facility location problems. Demand, travel time, facility costs, and even distance may change and these changes may occur in a random fashion. According to Owen and Daskin (1998), stochastic location problems can be broken down into two approaches: the probabilistic approach, which considers the probability distributions of the modelled random variables, and scenario planning approach, which considers a generated set of possible future values. In both cases, any number of system parameters might be taken as uncertain, including travel times, construction costs, demand locations, and demand quantities. The objective is to determine robust facility locations which will perform well under a number of possible parameter realisations

Probabilistic models extend deterministic location-allocation models in order to consider uncertainty and congestion of servers. Earlier work in this subject has been done by Daskin (1983) with the **maximum expected coverage location problem (MEXCLP)**, an extension of the MCLP in which facilities are assumed to be busy with probability p. The objective of this model is to maximise the demand that is covered by an available (i.e., not busy) facility. It is assumed that the probability of a facility being busy is independent of the probability of another facility being busy, and that this busy probability is the same for all facilities. The **maximum availability location problem (MALP)** allocates a limited number of servers in such a way as to maximise the population with a server available within the time/distance standard D with reliability α (ReVelle and Hogan, 1989a). The MALP has been further extended by different authors. Typical applications of the MALP and other extensions include the location of emergency facilities (Current and O'Kelly, 1992), the design of hierarchical health care systems (Moore and ReVelle, 1982; Mitropoulos et al., 2006) and the design of congested service systems (Marianov and Serra, 2001). An alternative to the MALP when the number of service facilities is not fixed is the **probabilistic location set covering problem (PLSCP)** proposed by ReVelle and Hogan (1989b) which seeks to minimise the number of servers needed to guarantee a predetermined minimum coverage requirement with a certain reliability. The PLSCP and other models have been combined with results from queuing theory to examine additional aspects of facility location. The extension of the PLSCP becomes the **queuing probabilistic location set covering problems (Q-PLSCP)**. The first one embedding queuing theory in facility location-allocation problems was Larson (1974) with the hypercube queuing model (HQM). The solution of this model provides state probabilities and associated system performance measures (e.g., workload, average service rate, loss rate) for given server locations. "The HQM is not an optimization model; it is only a descriptive model that permits the analysis of scenarios" (Galvão and Morabito, 2008). The first model proposed by Larson (1974) assumes that the service time is independent of the locations of the calls for service and the dispatched unit. This argument was supported by the idea that time spent on the road is negligible compared to service time. This can be a fact for fire brigades but not for the ambulances and on-demand vehicles. However, even with this simplification, the number of states grows exponentially as the number of servers is increased, which led Larson (1975) to also propose a heuristic method. Batta et al. (1989) used the model to show that the implicit assumption of server independence in Daskin's expected covering model is often violated. Berman et al. (1985) used an M/G/1 queuing model to explore the location of a single facility on a network as a function of the demand intensity when demands could wait for service. At very low and very high demands, they showed that the facility would be located at the 1-median location; for intermediate demand intensities, alternate locations were shown to be optimal. When no queuing was permitted, they showed that the optimal location was always the 1-median. There is a variety of more recent extensions of this work are: Atkinson et al. (2008) assume different service rates for each server in the system with equal interdistrict or intradistrict responses; Iannoni and Morabito (2007) and Iannoni et al. (2008) embed the hypercube in a genetic algorithm framework to locate emergency vehicles along a highway, extending the problem to enable multiple dispatch (e.g., more than one server can intervene for the same incident); Geroliminis et al. (2009, 2011) integrate the location and distracting decisions in the same optimisation and solve the problem by using steepest descent (Geroliminis et al., 2009) and genetic algorithms (Geroliminis et al., 2011); Boyaci and Geroliminis (2012) propose two new aggregate models for emergency response systems and other service on-demand vehicles which group servers into bins of servers, dramatically decreasing the number of states and making HQM applicable for medium sized problems.

An alternative to the probabilistic approach for stochastic location problems is **the scenario planning approach**. Scenarios represent possible values for parameters that may vary over the planning horizon. The objective is to find solutions which perform well under all scenarios. In some applications, scenario planning replaces forecasting as a way to evaluate trends and potential changes (Mobasheri, 1989). Decision makers can develop strategic responses to a range of environmental changes, preparing themselves for the uncertain future. Under such circumstances, scenarios are qualitative descriptions of plausible future states, derived from the present state with consideration of potential major industry events. In other applications, scenario planning is used as a tool for formulating and solving specific operational problems (Mulvey, 1996). While scenarios here also depict a range of future states, they do so through a quantitative characterisation of the various values that the problem input parameters may take. There are at least three approaches to incorporating scenario planning into location modeling (Owen 1998): (i) optimising the expected performance over all scenarios; (ii) optimising the worst-case performance; and (iii) minimising the expected or worst-case regret across all scenarios. A full review of the scenario planning approach can be found in Owen (1998) and Current et al. (2001).

3.3.1.4 From point like service area definition to more complex shapes: integration with ICT tools

For simplicity of problem formulation and solution, applications of the problems previously discussed usually assume that both the candidate facilities and demand areas are discrete point locations (Miller, 1996; Church, 1999; Farhan and Murray, 2006), where accessibility and spatial characteristics of the area, such as the existence of barriers or closeness to a dangerous area are not taken into account. More realistic problems can be formulated by considering space properties and the continuous nature of space. In this sense two different problems arise: one is related to the representation of the demand areas and the other to the spatial representation where potential facilities can be located. In this regard, location models can be categorised into two types: discrete or continuous problems.

Discrete problems

Murray and O'Kelly (2002) highlight representation issues in coverage modeling, showing that reliance on a point based abstraction of a region could lead to an overestimation of actual coverage. It has largely been recognised the need of coverage modeling approaches capable of addressing serving point, line and polygon demand areas (Miller, 1996; Benveniste, 1982). Line demand object applications include street or river segments, and polygon demand objects could be watersheds, census tracts or planning districts. Different solutions have been proposed using the help of Geographical information Systems where spatial features are represented as vector objects, to abstract demand areas as point, lines or polygons (Tong, 2007; Murray and Tong, 2007; Murray et al., 2008; Alexandris and Giannikos,2010).

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Starting from the point-like coverage definition, the following coverage properties have been proved for extended coverage areas (Tong, 2007; Murray and Tong, 2007): if a point location A is covered by a facility and location B is also covered by the same facility, then any point located in the segment AB is also covered, i.e coverage of a line segment is guaranteed when both end points are covered. For a polygon object with vertex, let's say, at ABCD, if each line segment end point in ABCD is covered, then any point on the boundary or within the polygon is also covered by the facility.

The coverage of object-like demand areas given previously is based on the definition of coverage of point like areas and it works well when the demand object is well inside the covering area. This definition is a bit crude since it implies that demand points at distance $\mathrm{D} - \varepsilon$ from a server are fully covered whereas points at distance $\mathrm{D} + \varepsilon$ are not covered at all, where $\varepsilon > 0$ may be arbitrarily small. Various approaches have been proposed to address this issue, where relaxation criteria to consider coverage should be applied. They can be summarised in a new classification of problems (Jabalamelia et al., 2011):

Gradual covering models: in this type of models, instead of all or nothing coverage, a coverage function is used. This function determines the proportion of demand covered at a specified distance from a facility. Church and Roberts (1984) expanded the notion of service coverage where the value of coverage is not a constant and formulate a set of models that are sensitive to many of the issues found in public facility location. They consider both convex and non-convex coverage functions and discuss their potential use in locating noxious activities. Berman and Krass (2002) proposed the network version of this problem with a step coverage function. Berman et al. (2003) define two distances, a lower distance and a larger one. Within the lower distance a demand point is fully covered and beyond the larger distance it is not covered at all. For a distance between these two values they assume a gradual coverage decreasing from full coverage at the lower distance to no coverage at the larger distance. Eiselt and Marianov (2009) extended the gradual covering problem in the framework of the set covering location problem. Berman et al. (2009) proposed the ordered gradual cover location problem (OGCLP) which combines the characteristics of gradual cover and ordered median models. Finally, Drezner et al. (2010) incorporated uncertainty in coverage radius in gradual cover models.

Cooperative cover models: when an individual coverage assumption is relaxed, all the established facilities may have an effect on coverage of a demand point. By exploiting the capabilities of Geographic Information Systems, Alexandris and Giannikos (2009) proposed a model that takes into account partial covering of demand areas by several servers. An area is considered adequately covered if it is fully covered by one facility or by several facilities providing partial coverage. Consider a case where each facility emits a signal. The amount of signal received by each customer from each facility is determined based on the distance between the facility and the customer. Thus, the total amount of signals received by each customer is the sum of signals received from all of the facilities. A customer is considered covered if the total amount of received signals is greater than a predefined threshold. Berman et al. (2010) proposed this concept for both maximal covering location problems and location set covering problems for the Euclidean distance case.

Variable radius models: in these models, the coverage radius of a particular facility is considered as a function of the establishing cost of the facility. The higher the establishing cost, the greater the coverage radius of the facility. Berman et al. (2009) introduced this idea to determine the locations, the number and the coverage radius of each facility to cover all the demand points with the minimum locating cost.

Continuous demand coverage

In some real world situations, demand is present everywhere within a region. Hence object-like definition of demand areas is not sufficient to ensure coverage. That is the case of emergency warning sirens, for instance: it is important for sirens to be audible in residential and commercial areas, but schools, places for outdoor recreation, transportation corridors and anticipated development areas may be equally important to serve (Current and O'Kelly, 1992; O'Kelly and Murray, 2004). The problem to solve is to locate a single facility in continuous space to maximise coverage of continuously distributed demand.

In the case of continuous demand, a common approach is to divide the demand area into sub-regions and select a single point out of each sub-region, which could be the centre of the sub-region, one of its vertices if the sub-region is represented by a polygon, a point selected randomly within the sub-region, etc. In this way the continuous demand is transformed into a discrete set of demand points and the MCLP can be applied to determine the locations of the servers. However the solution of the MCLP is not enough when the facility location is influenced by topographical barriers, as it is the case with sirens or telecommunication transmitters. An extension of the MCLP has been proposed to take into account the accessibility to the possible facility location. The maximal service area problem (MSAP) makes use of (GIS-enabled) network analysis — which takes into account network attributes such as road width, speed limit, barriers, turn restriction and one way restriction — to generate service areas of facilities as travel time zones. The travel time zone, in some texts referred to as travel time band, is a polygon layer, overlaid on the network. The MSAP is designed as a discrete model where a specified number of facility sites that optimise the objective function are selected out of a finite set of potential sites. The continuous space of the study area is deemed as the demand region. To simplify mathematical modeling, this continuous space is divided into discrete points. The optimisation problem is defined as the maximisation of the number of demand points intersecting with service area polygons of a set of facilities, i.e., a set of sites for the facilities is selected in such a way that the combined service area polygons of these sites encompass the largest possible area in the demand region. The number of demand points intersecting with the service area polygons acts as the surrogate information to measure the coverage area. An alternative approach to the discretisation of demand areas is that where the whole demand area is "mapped" into a 1-D object known as the medial axis (Timothy at al., 2009). The mapping of the demand areas is based upon the geometrical properties of a region assessed using GIS.

Continuous facility location

Facility location can be considered discrete or continuous. A continuous problem allows potential facility locations to be anywhere in the plane or a subset of the plane, whereas a discrete problem confines potential facilities to some finite set of sites specified in advance (Love et al., 1988). If there is only a finite number of feasible areas for locating service facilities due to costs, site availability or others reasons, then a discrete approach makes sense (Murray and O'Kelly, 2002). However, if any place can be a potential facility site in the region, a continuous approach is more suitable, particularly with respect to optimality. That is the case of alarm sirens, mobile phone towers and other facilities that can be located pretty much anywhere.

Siting facilities in continuous space in order to maximally cover discrete demand locations has been addressed by various authors (Mehrez and Stulman, 1982; Mehrez, 1983; Mehrez and Stulman, 1984; Church 1984), through what is known as planar maximal covering location problem (PMC). In order to solve the PMC, properties of coverage are exploited to discretise this continuous space problem. To identify candidate facility locations in the continuous domain, the geometry of the location problem is used to identify a set of potential locations that will contain optimal facility sites. This set of candidate locations is formed by finding the intersection of all circles of radius S centered at each discrete demand location. These points are referred to as the circle intersection point set (CIPS) (Church, 1984).

Tong (2007) considers not only point base demand problems but more complex objects (points, lines and/or polygons), covered by a specified number of facilities located in continuous space assuming a general distance metric. This extension is known as is the extended planar maximal covering location problem (EPMC). To solve the EPMC problem the infinite number of possible locations is reduced to a finite set of critical locations. These are identified using the so-called polygon intersection point set (PIPS) approach: (i) identify spatial demand objects (points, lines and/or polygons) in need of coverage; (ii) extract object vertices as potential facility locations, (iii) derive covering boundaries (areas) for each demand object; (iv) identify the intersection points of covering boundaries as potential facility locations; and (v) remove dominated critical locations (optional). The extraction of these locations is supported by standard commercial GIS functionality. Step 5 recognises that the number of critical locations identified may be rather large and that some locations may in fact be better than others in the sense that one location can cover everything that another location can cover and more. This domination principle has long been recognised in coverage modeling (Toregas and ReVelle, 1973).

3.3.2 Top-down static models

This category corresponds mainly to what are known as spatial interaction models. The precursors of the spatial interaction models, Alonso bid model and the Lowry model, are also in this category. Although these models are out of use, their basic principles are incorporated into more sophisticated urban models.

3.3.2.1 Alonso's bid model

The Alonso bid model is based on urban economic principles. It predicts the change of the prices and demand of real state according to its distance to the central business location. According to Alonso's bid theory, different land users compete for the land close to the city centre, which is assumed to have greater access to cultural and business opportunities and hence to be the most expensive. This theory is based upon the reasoning that the more accessible an area is, the more profitable it is. Residents make bidding choices that maximise their utilities under the trade-off between commuting and housing costs. The amount they are willing to pay is called "bid rent". The result is a pattern of concentric rings of land use, creating the concentric zone model in a declining trend of population density, land value, and housing price with distance from the CBD (Anas et al, 1998; Parker and Filatova, 2008). Although more sophisticated models have been proposed after Alonso's ideas, the Alonso's bid theory is still integrated in many of the modern urban models. Analytical extensions of the original model have been developed by incorporating open-space amenities and spatial externalities (Caruso et al, 2007; Cavailhès et al, 2004; Irwin and Bockstael, 2002; Wu and Plantinga, 2003). This field has developed further to create a polycentric extension of the original monocentric city model (Fujita and Ogawa, 1982; Fujita and Thisse, 2002; Harris, 1985; Munroe, 2007; Ogawa and Fujita, 1980).

3.3.2.2 The Lowry model. Spatial interaction models

The model developed by Lowry in 1964 was the first attempt to implement the urban land-use transport feedback cycle in an operational model and it is still the most widely applied of all operational urban models (Batty, 2009). The Lowry model is a model of the evolution and distribution of urban land use. It is based on the assumption that activities can be predicted from a given level of basic employment. The model uses economic based theory to determine the total population and employment in service industries. The population is assigned to zones of the city in proportion to their population potential. Service employment is allocated in proportion to the market potential of each zone. These allocations have to meet land use constraints, notably housing densities, and threshold populations for the various services. The model is run several times while allocations are determined until the system reaches equilibrium. The original Lowry model assumed the level of location of basic employment to be exogenous and computed the level of population and its dependent service employment using an economic-based model, and their spatial distribution using interaction functions, allowing policy simulations for different specified patterns of basic employment.

It can be said that most spatial interaction models are an extension of or belong to the family of Lowry type models. The main purpose of these models is to describe and help predict spatial flows of people, commodities, capital and information between two locations occurring in a given period of time. The length of the relevant period of time is determined by the kind of flows under study, which also define the relevant actors and important locations (Fischer, 2004). Interaction between important locations and relevant actors are represented by an interaction matrix

$$\mathrm{T}_{\mathrm{IXJ}} = \begin{bmatrix} \mathrm{t}_{11} & \cdots & \mathrm{t}_{1\mathrm{J}} \\ \vdots & \ddots & \vdots \\ \mathrm{t}_{\mathrm{I}1} & \cdots & \mathrm{t}_{\mathrm{IJ}} \end{bmatrix},$$

where I is the total number of origins and J the total number of destinations, and the t_{ij} elements of the matrix represent the interaction between the origin i and the destination j. The total outflows from origin i are given by the sum of the i^{th} row of the matrix, while the total inflow of each of the destinations j are given by the sum of the j^{th} column.

3.3.2.3 Gravity type models

Gravity type models are spatial interaction models where flows occurring between two zones are determined by three factors: (i) the distance between the two locations, (ii) the origin propulsiveness and (iii) the destination attractiveness. The distance between two locations, which does not necessarily have to be a physical distance (e.g. it can be cultural distance), follows an inverse relationship with the expected flow known as a distance decay or distance deterrence relationship (Fischer 2004). Origin propulsiveness refers to the ability of origins to generate flows. There is a positive relationship between the volume of flow generated from an origin and some appropriate measure of propulsiveness. The same applies to the zone attractiveness, which refers to the ability of a zone to attract flows.

The basic structure of the spatial interaction problem is, then, to express the volume of interactions in terms of origin, destination and spatial separation factors. In the traditional **gravity model**, inspired by the Newton's gravity law, the deterrence function is specified by a power law, and flows between two zones are directly proportional to the number of actors leaving the origin zone (origin propulsiveness) and the number of actors entering a destination zone (destination attractiveness):

$$t_{ij} = b\, r_i\, s_j\, \left(C_{ij}\right)^{-\alpha}$$

t_{ij} represent the flows between zones i and j, b is a proportionality constant, r_i represents the propulsiveness of the origin i, s_j the attractiveness of the destination j and C_{ij} is a measure of separation or interaction costs. The proportionality factor b and the value of the power α are fitted with empirical data.

In principle the traditional gravity model does not consider any conservation constriction over the total number of outflow from one zone and the total actors leaving that zone, and the same for the total inflows. Several authors have taken the gravity model as a starting point and explored the effect of imposing conservation conditions over out and in flows as well as of considering different definitions of the deterrence function. This has given rise to a whole family of models known as gravity type models. The general mathematical expression for these models is:

$$t_{ij} = b_{ij}\, r_i\, s_j\, f_{ij}\,,$$

where the deterrence function, f_{ij}, can be specified by an exponential or a power law. These two specifications can be combined into a more flexible two-parameter family of gamma deterrence functions for any positive scalar measure of separation or interaction costs, C_{ij}:

$$f_{ij} = f\left(C_{ij}\right) = \left(C_{ij}\right)^{-\alpha} \exp(-\beta C_{ij})$$

Three different variants of this model can be obtained by imposing constraints over the flows. The **production constrained case** assumes that the total number of actors leaving each zone, i, is known, i.e $\sum_j^J t_{ij} = r_i$ where r_i is known, imposing a condition over the proportional factor.

$$b_{ij} = b_i = (\sum_j^J s_j\, f_{ij}\,)^{-1}$$

In the **attraction constrained case** the total number of actors entering each zone, j, is assumed to be known, i.e. $\sum_i^I t_{ij} = s_j$. The **double (production–attraction) constrained case** assumes that both the total number of actors leaving a zone i and the total number of actor entering a zone j is known. It is also assumed by simplicity that $b_{ij} = b_i\, b_j$ (Wilson, 1967), with

$$b_i = (\textstyle\sum_j^J b_j\, s_j\, f_{ij}\,)^{-1}\,,$$

$$b_j = (\sum_i^I b_i\, r_i\, f_{ij}\,)^{-1}$$

Each b_i is dependent on all the b_j and vice versa and hence the previous equations are solved iteratively.

Although calculated from a different approach, the **entropy maximising model** is a special case of the double constrained case. The deterrence function in this case is specified by an exponential:

$$t_{ij} = b_i\, b_j\, r_i\, s_j\, \exp(-\beta C_{ij}).$$

The entropy maximising model was deduced by Wilson (1967) from a probabilistic approach based on statistical equilibrium concepts from statistical mechanics. Considering all the possible configuration of flows between pairs of locations, one extreme configuration is that where all origins end up in only one destination, and the other extreme is that in which the inflows are evenly distributed between all destinations. Between these two possible configurations there is a whole range of possible flow distributions. The number of possible configurations are defined by:

$$W = \frac{(\sum_i^I \sum_j^J t_{ij})!}{\prod_j \prod_i t_{ij}!}$$

According to Boltzmann result in statistical mechanics, one of these states is overwhelmingly the most probable, subject to any constraints that have to be satisfied. Such state is obtained by maximising W. This is equivalent to maximising log(W), which makes it more tractable mathematically. Assuming that the number of states is large and that not all of them are equally probable, by applying the Stirlin's rule one obtains

$$W = -\sum_i^I \sum_j^J t_{ij} \log(t_{ij}),$$

which looks like Boltzman's definition of entropy for the micro-canonical state in statistical mechanics. The entropy maximising model calculates the most probable configuration under the conditions of the double constrained case plus the extra constraint that the total transport cost is constant, i.e.

$$\sum_{i,j}^{IJ} t_{ij}\, C_{ij} = C,$$

Under such constrains it is found that the most probable state is that which accomplishes

$$t_{ij} = b_i\, b_j\, r_i\, s_j\, \exp(-\beta C_{ij})$$

with

$$b_i = 1/\sum_k^K b_k\, s_k\, \exp(-\beta C_{ik}) \quad ,$$

$$b_j = 1/\sum_k^K b_i\, r_k\, \exp(-\beta C_{kj}) \quad .$$

where β is the Lagrangian multiplier resulting from the maximisation. As in the case of gravity models the value of β is also fitted with empirical data. For a detailed deduction and discussion of the entropy models see Wilson (2010).

Specific sectoral applications

Travel cost and the attractiveness of a zone are defined depending on the kind of flows under study. Gravity and entropy-maximisation models have been widely use to describe commuting or migrations flows. In such cases the attractiveness of a zone is defined as the number of job opportunities and/or the range of salaries available and the propulsiveness of a zone as proportional to the population density; the distance or travel cost is defined as a function of physical distances and accessibility.

A variation of the entropy maximising model has been used as a location model for the retail sector, where retail can be interpreted very broadly: any system of interest where there is a flow from a population area to some kind of facility. In this case a new variable has been introduced, x_j, the resident's (shopper's) benefit gain from buying in a zone with a particular size, which is proportional to the zone attractiveness W_j. The total benefit $\sum_j^J x_j = X$ is assumed to be conserved:

$$\sum_{i,j}^{IJ} t_{ij} \log W_j = \sum_{j}^{J} s_j \log W_j = X$$

The propulsiveness of a zone I is defined as the total population at the zone, P_i, times the average expenditure per capita at the zone e_i. The total amount expended by zone i is then a conserved quantity, $\sum_j^J t_{ij} = P_i\, e_i$. The most probable configuration is then calculated by maximizing $-\sum_i^I \sum_j^J t_{ij} \log(t_{ij})$ subject to the total outflow, total benefit and total travel cost constraints. The result of this maximisation gives:

$$t_{ij} = b_i\, P_i\, e_i\, W_j^{\alpha} \exp(-\beta C_{ij}),$$

with

$$b_i = 1/\sum_{k}^{K} W_k^{\alpha}\ \exp(-\beta C_{ik})$$

where α and β are the Lagrangian multipliers of the constrained maximisation. W_k^{α} can be interpreted as the attractiveness of the zone and hence the term $W_k^{\alpha}\ \exp(-\beta C_{ik})$ can be seen as the pulling power of zone j over zone i and $(b_i)^{-1}$ would be the competition of other "commercial" centres.

This model can be disaggregated to consider different type of retail zones, h, type of good, g, and socioeconomics profiles, n:

$$t_{ij}{}^{ngh} = b_i{}^{ngh} P_i{}^{ng} e_i{}^{ng}\, W_j^{gh} \exp(-\beta C_{ij})$$

In general it is assumed that W_j is proportional to the shopping centre's size (Wilson, 2008), but other parameters such as number of employees or variety of business offer can also be considered as a measure of attractiveness.

Disadvantages of gravity type models

Historical data is needed to calibrate these models, making them useless in new zones where such data does not exist. Also due to the lack of time dependence and of a behavioural framework, no causal relationships can be established.

Masucci et al. (2013) have shown that these models (as well as the radiation model discussed below) fail to accurately predict mobility patterns within London. They argue that this is due to the lack of explicit consideration of the sociogeographical segregation phenomena (Theil and Finizza, 1971) and residential/business ward specialization (Simini, 2012) which are key drivers in determining the structure of flows and the density of population in the city.

3.3.2.4 Radiation model

Another spatial interaction model is the radiation model proposed by Simini et al (2012) to solve the inconsistencies of the gravity model: the lack of a rigorous derivation of the expression; the lack of theoretical guidance for adjusting the deterrence functions and the large number of parameters to fit the empirical data; the need for previous data; and the predictive discrepancies found in the gravity model.

The radiation model is inspired by the particle diffusion model, where particles emitted at a given location are absorbed by the surrounding location with a probability $p(z)$. Each individual seeks a job offer in all zones including its own residential zone. The job offer should accomplish with the minimum standards to be attractive for the individual (which in radiation theory is known as the absorption threshold). The benefits of a potential employment opportunity, z, are randomly chosen from distribution p(z) where z represents a combination of income, working hours, conditions, etc. Each zone has a number of employment opportunities, which is proportional to the resident population, n, assuming that there is one job offer for every n_{jobs} individual (capacity of absorption). Job offer may have different benefits, z, and so, each zone with population n is assigned with random numbers, $z_1, z_2, \dots, z_{[n/njobs]}$, chosen from a distribution p(z),representing different job options in terms of benefits (absorbance capability). The individual chooses the closest job to his/her home whose benefits z are higher than its "expectations" (absorption threshold) and higher than the best offer available in his/her home zone, prioritising non-commuting over the benefits, i.e. individuals are willing to accept lesser benefits closer to their home. This process applied to each resident of each zone determines the daily flows t_{ij} between zones.

According to the radiation model, the probability that a particle (outer commuter) of a zone i with total population m_i gets absorbed (finds a job) at a zone j located at an r_{ij} distance is given by

$$P(1|m_i, n_j, s_{ij}) = \int_0^\infty dz\, P_{m_i}(z) P_{s_{ij}}(< z) P_{n_j}(> z),$$

where s_{ij} is the population density in a circle of radius r_{ij} (excluding the population of i and j), $P_{m_i}(z)$ is the probability that the maximum value extracted from $p(z)$ after m_i trials is equal to z, $P_{s_{ij}}(< z)$ is the probability that s_{ij} numbers extracted from the $p(z)$ are less than z. This means that the probability of commuting to zone j (choosing a job in zone j) is equal to the probability of not finding a job, within your expectations, in your zone, times the probability of not finding a job, within your expectations, in the surrounding areas (circle of radius r_{ij}) times the probability of finding a job, within your expectations, in j.

By solving the above integral one obtains

$$P(1|m_i, n_{j,}s_{ij}) = \frac{m_i n_j}{(m_i+s_{ij})(m_i+n_j+s_{ij})}.$$

Notice that the above expression is independent of $p(z)$. The total flow between two zones is then given by

$$\langle t_{ij} \rangle = t_i \frac{m_i n_j}{(m_i + s_{ij})(m_i + n_j + s_{ij})}$$

where t_i is the number of particles (outer commuters) emitted by (leaving) zone i (Simini 2012). This result means that the probability of having exactly t_{ij} commuters between zones ij depends only on the origin population, the destination population and on the population sij in a circle of radious r_{ij} equal to the distance between origin I and destination j and centred at the origin i.

Disadvantages of radiation models

Simini et al. (2012) claim that the radiation model improves the flow predictions between zones by solving the inconsistencies of the gravity model and thanks to the independence from metrical distances and its parameters-free nature. However, in a comparative analysis performed by Masucci et al. (2013), both gravity and radiation models have a poor performance describing transportation data. Even when the parameter-free formulation of radiation models presents an advantage over the gravity models in cases where there is no data available, they share other characteristics with the gravity model, such as the lack of behavioural information and temporal dependence.

3.3.2.5 Utility maximising models

An alternative derivation of the entropy maximising model is the utility maximising approach. The essential idea of this approach is to include the microeconomic paradigm of random utility maximising choice behaviour into spatial interaction models. A utility function that represents the net benefit of making a trip from *i* to *j* for a given purpose is defined, and it is assumed that the destination location chosen by an individual corresponds to that which maximises its utility function. This can be done by calculating an average benefit v_{ij} and adding a random component, ε_{ij}, reflecting different perceptions in the population. The utility u_{ij} from an individual resident in *i* selecting destination *j* will then be given by

$$u_{ij} = v_{ij}(z_{ij} + \alpha) + \varepsilon_{ij}$$

where z represents a vector characterising the choice-destination *j* and its separation from *i,* and α is a vector of parameters in the utility function. This utility cannot be predicted with exactitude for each individual, due to their random component, but a probabilistic statement based on the individual's discrete likelihood of choosing a particular destination can be derived (Ben-Akiva and Lerman, 1985). The definition of the functional forms v_{ij} and ε_{ij} have given rise to a whole family of models. The most commonly assumed probability is the Weibull distribution which results into the multinomial logit model developed by McFadden (1973):

$$p_{ij} = \frac{\exp(\mu v_{ij})}{\sum_{k=1}^{J} \exp(\mu v_{ik})}$$

It is interesting to note that random utility maximising models are traditionally estimated from disaggregate data, while gravity and entropy maximising models are generally estimated from aggregate data.

Competing destination models

Competing destination models are an alternative of utility maximising models where the individuals' limited capacity for information processing is recognised. In the competing destination model, proposed by Fotheringham (1983), it is assumed that spatial information is processed hierarchically and that clusters of destinations are initially evaluated. It is further assumed that individuals underrepresent the magnitude of large clusters so that the probability of selecting an individual destination within a large cluster is lower than if the destination were relatively isolated. This implies that the probability of selecting alternatives in large clusters will be lower than if spatial information were processed non-hierarchically and all destinations were evaluated. The competing destinations framework incorporates this effect by the addition to the standard multinomial logit model framework of a variable which measures the centrality of a destination (i.e., the likelihood of a destination being within a large cluster). This variable can be measured in different ways, but an obvious one is the well-known potential accessibility measure (Pellegrini, 1999):

$$C_j = \frac{1}{J-1}\sum_{j' \neq j} \frac{W_j}{d_{j'j}}$$

WhereW_j represents the size of the place *j* and $d_{j'j}$ represents the distance between alternative shopping areas *j'* and *j*. With the inclusion of this new variable the probability for a user to choose a given location *j* is given by

$$p_{ij} = \frac{C_j^{\theta} \exp(\mu v_{ij})}{\sum_{j=1}^{J} C_j^{\theta} \exp(\mu v_{ij})}$$

where θ is a parameter that relates the probability of choosing an alternative to the spatial competition faced by that alternative. It also acts as an index of the intensity of hierarchical choice. For θ=0, the competing destinations framework collapses to a traditional multinomial logit model. C_j is the competition variable and v_{ij} the utility of zone *j* for an individual coming from zone *i*. It is expected that θ will be negative, since for $\theta < 0$ the probability of being selected decreases as the competition variable increases, while for $\theta > 0$ the probability of being selected increases with the competition.

Disadvantages of utility maximising models

Specific disadvantages of the utility maximising models are that the abstract concept of "utility", which the locator seeks to maximise, cannot be directly measured, and that the Weibull distribution produces a conveniently simple model, but there is not much evidence that real distributions of stochastic terms are of that form.

3.3.3 Top-down non-spatially explicit dynamic models

3.3.3.1 Boltzmann-Lotka-Volterra

The Boltzmann-Lotka-Volterra model is actually a hybrid of a static model plus a dynamic module that introduces the time dependence of the system. The fast dynamics, e.g. the individual movements between zones at each daytime, is averaged out (e.g., using the entropy-maximising model) and assumed to be in equilibrium. The slow dynamics, i.e. the eventual changes in the attractiveness or propulsiveness of a zone, is taken into account by adding a time dependent factor in the model.

If the inflows s_j are equal to the attractiveness W_j of the zone j, the system is in equilibrium and no changes occur. However if the inflows are larger than the total benefit of the zone, i.e. if $s_j < K\mathrm{W}_j$, where K is a proportionality constant, then the centre should grow; on the contrary if $s_j < K\mathrm{W}_j$ the centre is likely to decline (Wilson, 2008). These changes on time can be modelled by adding the multispecies Lotka-Volterra model to the static model:

$$\Delta \mathrm{W_j}\,(\tau, \tau + 1) = \varepsilon\left[\mathrm{s_j} - \mathrm{KW_j}\right]\mathrm{W_j}$$

where ε is a parameter that measures the strength of the response and ΔW_j represents the change of attractiveness of zone j. In the case of a retail model, for example, if $\mathrm{W_j}$ is approximated by the area of the commercial floor space, then this means that the size of the commercial area will increase either by increasing the number of businesses or by increasing the size of those already existing.

In the most general case, the equilibrium state can be calculated using different models. Also the dynamical model can be approximated by other kind of population models such as prey- predator or competition models.

3.3.3.2 System dynamics models for land use

In contrast with the top-down models discussed previously, models based on a system dynamics approach consider not only interactions between different elements of the system, but also the effect that an alteration of one of the elements will have on the future dynamics of the others. Real world processes are represented in terms of stocks (e.g., material, knowledge, people, money…), flows between these stocks, and information that determines these flows. The objects being modelled — people, products, events, and other discrete items — are considered at a high degree of aggregation. The system behaviour is defined as a number of interacting feedback loops, balancing or reinforcing. Feedback loops represent the effect of an alteration in any variable of the systems on the other variables. Feedback loops also take into account delayed structures since the effect of a change of a variable on the others may be delayed in time.

Specific sectoral applications

In the context of land use and urban development, system dynamics models have been used for studies of sustainable development (Shen et al., 2009; Xu and Coors, 2011; Bottero and Mondini, 2003; Brandon and Lombardi, 2005; Nessa and Montserrat, 2008), considering different dimensions such as the housing sector, economy, environment, etc. The essential characteristic of these models is the time iterative process occurring within and between the sub-models modelling the dynamics of each element of the system. These together with the ability of integrate different dimensions (economic, environmental, social, demographics, etc.) make them a good candidate for policy testing and what-if scenarios' evaluation, as long as the individual characteristics of the elements are not essential for the dynamics.

Disadvantages of system dynamics models

The main disadvantages of these models are the high degree of aggregation and the lack of spatially explicit characteristics included as endogenous variables of the model.

3.3.4 Top-down spatially explicit dynamic models

These models correspond to models based on system dynamics approaches where spatial characteristics of the models are explicitly taken into account and are used to approximate economic parameters. The common models falling in this category are the spatially explicit econometric models, where one of the sub-systems taking part of the feedback loop represents the dynamics of economic parameters, usually extracted from spatial information.

3.3.5 Bottom-up dynamic models

In bottom-up models, the agents and the interactions between them are central to the simulation of emergent behaviours. Bottom-up models are mainly based on decision rules in which agents behave in response to their environment and to other agents.

3.3.5.1 Cellular automata models

In **cellular automata (CA) models**, the system is divided into discrete, usually (although not necessarily) regular zones of physical space called cells. The size of the cell should be small enough so that individual space characteristics are reflected. Each cell has a value representing the current state of the "territory". States can be (i) binary values (e.g., urban/non-urban or occupied/empty space), (ii) qualitative values that represent different land uses, (iii) quantitative values that represent, for example, population density (Li et al., 2003), degree of development (Yeh and Li, 2002) or the value of buildings (Cecchini and Rizzi, 2001), or (iv) a vector of several attributes (Portugali and Benenson, 1995). Urban growth or future urban patterns are predicted or modelled by the evolution of the systems according to cell's stage transitions. Transition rules can range from simple state transition models, in which cells change states (land uses) according to some observed probability, to the more general form of CA, in which the state of a cell depends on its previous state and on the state of its neighbouring cells. In the more general case, and according to the problem, CA models establish the shape of the cells grid; the maximum distance of influence between cells (when it is extended beyond the nearest neighbours, a distance-decay effect is usually introduced); the compatibility between land uses; the influence of socioeconomic environmental or sustainability factors in a land conversion process, e.g. as in Webster and Wu (2001) or Caruso et al. (2007), where transition rules are based on microeconomic theories of planning; and the spatial characteristics to be considered, such as territory orography, distance to a lake, accessibility to an area by public transport, etc. Regarding time dependence, the transition rules can be modified every time step according to changes in configuration and external parameters. A full review of the use of CA models for urban processes can be found in Batty (2005) and Santé et al. (2010).

Granularity properties of CA

The level of temporal and spatial aggregation of CA models tend to be quite flexible, although for urban and land cover systems, the spatial resolution is often land parcels and census tracts and above, while temporal intervals are often one year periods. CA can be easily integrated with Geographical Information Systems (Itami, 1994; Wagner, 1997) and, consequently, build models at high spatial resolution.

Disadvantages of CA models

CA models do not strictly conserve quantities of population, as there is nothing intrinsic to the model that limits growth or decline, although models are often subject to macro-constraints provided by other models in their wider environment. The feedback in space is simplistic and often unrealistic in that the CA nearest neighbour influence principle, which is essential for physical diffusion processes, is not a good analogue for certain spatial effects where there is action-at-a-distance. Finally, CA models are not based on socio-economic foundations, but they are essentially “physicalist”, in the sense that they do not embrace detailed demographics.

3.3.5.2 Agent-based models

A more complex and sophisticated kind of dynamic bottom-up models are **agent-based models (ABM)**. These models consider not only space evolution and spatial interactions but also individual actors interacting with the environment and between them. Agent-based models consists of autonomous decision-making entities (agents) capable of processing information and exchanging it with other agents, an environment through which agents interact, rules that define the relationship between agents and their environment, and rules that determine the sequencing of actions. Autonomous agents are composed of rules that translate both internal and external information into internal states, decisions, or actions. Agents may or may not be spatially explicit. In contrast to CA models, agents tend to be mobile in a spatial sense and even if they do not physically move in space, they can be associated to different spaces and their change over time can reflect an implicit process of movement.

As in CA models, the agents’ characteristics and interactions rules are defined depending on the case of study. Agents may have goals and an awareness or sense of their surroundings. They can also be supplied with prior knowledge of their environment and awareness of other entities, obstacles, or required destinations. They can learn and adapt their behaviour. The relationships linking agents to other agents and/or other entities within a system may be specified in a variety of ways, from simply reactive (i.e., agents only perform actions when triggered by some external stimulus, e.g. actions of another agent) to goal-directed. The behaviour of agents can be scheduled to take place synchronously (i.e., every agent performs actions at each discrete time step), or asynchronously (i.e., agent actions are scheduled by the actions of other agents, and/or with reference to a clock). Thus, agent-based models offer a high degree of flexibility that allows researchers to account for heterogeneity and interdependencies among agents and their environment.

In the context of land use, an agent may represent a land manager who combines individual knowledge and values, information on specific land characteristics (e.g., orography, accessibility, etc.) and an assessment of the land management choices of neighbours (the spatial social environment) to make a land-use decision. The agents may also represent higher-level entities or social organisations such as a village assembly, local governments, or a neighbouring country. Interaction rules may include decision rules of each actor, such as income maximisation or minimum subsistence levels, their environmental feedbacks, and carry-over of spatially distributed resources. The environment may be defined by landscape, land markets, social networks, and resource management institutions. Most of the applications of ABM to land use are in the form of LUTI models which describe the relation between housing and job relocations and daily mobility patterns (Huang, 2013; Parker, 2002). Mobility patterns in these models are considered to be generated by the accomplishment of daily agents' activities. There are also some examples of the use of ABM for retail locations, as in Birkin and Heppenstall (2011). A full review on ABM can be found in Heppenstall et al. (2012).

Granularity

In terms of aggregation, agent-based models tend to be highly disaggregated down to the point where individuals constitute their basic units. They tend not to be constrained in terms of conserving any key quantity, although they may be structured to generate or conserve a certain level of population, especially if the focus is on movement in a fixed space as in pedestrian models.

Disadvantages of ABM

Agent-based model are expensive in computational terms and usually require more disaggregate data and more detailed sub-models to describe the behaviour of agents.

3.3.6 Bottom-up "static" models

Spatially-explicit econometric models can be considered as a special case of CA or ABM. **Spatially explicit land use models** (e.g., Hall et al., 1995) are cell-based models of land use which often use decision rules to describe the relationships between land use and human and biophysical factors. These decision rules are used to trigger land use changes. These models are distinguished by their conceptualisation of the conversion decision as an economic transaction where expected payoff must exceed costs. The models are developed by identifying the actors, conceptualising the drivers of their economic decision process, hypothesising the variables that reflect those drivers, and developing statistical approaches to test such hypotheses. Transition rules depend on some observed probability function dependent on the value of explanatory variables, which are spatially heterogeneous site and location characteristics. Models may be estimated in two stages where selling prices are used as an independent variable to characterise desirable parcel characteristics, and then the probability of conversion is modelled as a function of the selling price in the converted (developed) use, the costs of conversion, and the value of the land in its undeveloped use, such as agriculture or forestry (Bockstael, 1996). They are considered "static" models in the sense that the probability function is fixed and it does not get updated as the neighbouring cells change. Once the cells' state has been updated according to the transition rule, the system reaches an equilibrium state and becomes static.

3.3.7 Hybrids models

Hybrids models combining and integrating different modelling paradigms as sub-models of a full model have been proposed in order to take advantage of the specific properties of each of them, e.g. CA and ABM have been integrated with system dynamics models to account for the fast and slow dynamics occurring in a land use system (Lauf, 2012; He, 2005). Some examples of specific hybrid models are discussed in detail in section 5.

4 SWOT analysis. Usefulness and limitations of models for policy assessment

4.1 Optimisation-prescriptive location-allocation models

As previously discussed, optimisation location-allocation models are a prescriptive tool rather than a predictive/descriptive tool. They are useful to make decisions on the best allocation of facilities (e.g., retail services or public services) for given scenarios or for future possible scenarios. However, they do not take into account the effect that the allocation of a facility will have on the development of other sectors. They may take into account changes in demand, but they are not able to foresee how a given location geometry will influence changes in demand of that given facility and/or other. Hence they are not suitable for policy testing. However they may work well as tools to generate plausible scenarios of services location to feed into more sophisticated predictive/descriptive urban models. They are also very useful for services allocation under circumstances of budget and/or space scarcity. The stakeholder consultation conducted by INSIGHT shows that traditional location-allocation models are among the models most commonly used by cities (INSIGHT, 2014).

4.2 Top-down static models

Top-down static models, which mainly correspond to spatial interaction models are also among the most used models (INSIGHT, 2014). This is largely because they attempt to be comprehensive in simulating location and interaction, land use, and transport. But in this they sacrifice detail and process. They are largely non-dynamic, which limits their applicability (Batty, 2009).

Spatial interaction models are able to predict migration flows between zones based on population or job offers changes. At a certain extend they are able to explain drivers for land use change and travel patterns, as long as these are related with population migration or economic factors, e.g. the effect on mobility patterns due to a set of policies affecting the development of a commercial area can be evaluated with these models. They can be used to evaluate any kind of future scenarios where the main interactions to consider are flows between zones.

The kind and amount of data required by spatial interaction models is usually available in census and other official sources. However, since most of the model parameters are fitted using historical data, they often fail to to predict and/or explain changes due to new events and they cannot be used for places where no previous data exist. Also, due to their lack of time dependence and of a behavioural framework, causal relationships cannot be established.

These models are useful for short-medium term policy planning, such as transport network planning, provided that possible future scenarios are well known in one of the two main dimensions, origin/destination (population density/job offer or demand/service offer, etc.), considered by the models. These models are often used as sub-models of more complex land-use tools.

4.3 Top-down spatially and non-spatially explicit dynamic models

The inclusion of time dependence in top-down models allows foreseeing future land use scenarios at different time periods. Together with the integration of economic and geographical information as endogenous variables, in the case of spatially explicit models, this allows the evaluation of what-if scenarios where the secondary effects at later time steps on other sectors not directly affected by a given set of policies can also be evaluated. In the case of dynamic spatial interaction models, e.g. Boltzmann-Lotka-Volterra models, the effect of a policy affecting transport prices and/or distances on the future growth or decline of a commercial centre and the latter effect of this growth/decline on mobility patterns, for instance, can be evaluated. As for the case of system dynamics and spatially explicit models, they allow the introduction of more sophisticated parameters as endogenous variables, such as the influence of specific land attributes or economic aspects on the development of a business or residential zone. Still, due to the high level of aggregation and the lack of behavioural information, they may not capture the effects of certain policies on the dynamics of land use.

4.4 Bottom-up models

Bottom-up models consider individual interactions among different actors taking part in urban dynamics. They are flexible, particularly in relation to the development of geospatial models, and can be built even with very little knowledge of the global interdependences driving the system dynamics, based on knowledge/perception of local interaction rules. Their highly disaggregate nature and the rule transition approach under which they work allow the modelling of emergent properties. Behavioural rules help capture more complex structures and dynamics allowing the exploration of what-if scenarios under unexpected circumstances. Although they are not the most frequently models, they can help in the analysis of threats and opportunities during the process of policy making, by presenting possible scenarios as a consequence of the application of different sets of policies.

One of the main strengths of CA models is their capability of fast information processing and the illustrative nature of the results, which can be effectively interpreted by visual perception (Iltanen, 2012). Even when simulations do not necessarily represent the behaviour of real urban systems, they may reveal some essential mechanisms that are part of the overall dynamics. The models can be used as tools within urban planning to produce unforeseeable development paths and to help generate scenarios for the basis of decision making. CA models only consider the changes occurring within the urban structure (morphology) and even if these relations are in some indirect way dependent on economic and/or social aspects, these models do not contain explicit information about migrations patterns and/or socio-economic patterns and behaviour, and therefore they can only be used to evaluate policies affecting directly land use morphology. They can also be a sub-model of hybrid models, e.g. by combining them with system dynamics models (Laufa et al., 2012; Chunyang, 2005).

Due to their behavioural nature, ABM are a natural method for describing and simulating a system composed of real-world entities, at the same time that they provide a natural environment for the study of certain systems (Castle and Crooks, 2006). The versatility of ABM to represent as agents all kinds of elements of a system allows the testing of a given set of policies directly affecting a specific sector on the other elements of the system (including level of acceptance and other social reactions). The flexibility to tune the level of complexity and aggregation allows experimentation with aggregate agents, sub-groups of agents, and single agents, with different levels of description coexisting within a model.

One of the main problems for modelling design, calibration and validation is data availability (INSIGHT, 2014). In this sense, ABM has a big disadvantage since they need a large amount of data to be calibrated and validated. ABM requires the description of many agent attributes and behaviours, and their interaction with an environment. This can be tackled through multiple runs, systematically varying initial conditions or parameters in order to assess the robustness of results (Axtell, 2000) and this process can be extremely computationally intensive and thus very time consuming. Another disadvantage is that they are highly sensitive to initial conditions and to small variations in interaction rules (Couclelis, 2002), which together with the fact that the agents' heterogeneity is often introduced by local randomness, does not generate the sort of deterministic prediction that is usually needed for operational decision making (Batty, 2009).

5 Review of tools

There are almost as many urban modelling tools as groups working in the topic. In the stakeholder consultation performed by INSIGHT, most urban modellers involved in the process of policy making have declared to work with in-house developed tools (INSIGHT, 2014). However, most of these in-house tools are based on existing tools and already discussed modelling paradigms. In this section we review some tools built on one or more of the urban modelling paradigms discussed in the previous section. The tools which are candidates to be used in the project are reviewed in more detail, and other state-of-the-art tools are briefly discussed.

5.1 INSIGHT simulation tools

INSIGHT will adopt an eclectic approach, aimed at further developing a variety of tools based on different modelling paradigms. The candidate tools to be used in the project are the agent-based simulation frameworks UrbanSim and Albatross; the more aggregated, static framework Simulacra; MARS, based on a system dynamics model; and Metronamica, which implements a cellular automata model. The proposed strategy stems from the idea that there is not a 'one size fits all' modelling framework for all kinds of problems, but a set of modelling approaches with the potential to enrich our understanding or urban systems from different perspectives. These tools will be later on tested and evaluated through several case studies, so as to carry out a critical comparison of the advantages/disadvantages of different approaches to tackle various types of policy questions.

5.1.1 MARS

General information

MARS (Metropolitan Activity Relocation Simulator) is a system dynamics spatial interaction model designed to explore land use and transport interaction which can be used as a prescriptive or descriptive tool. It is mainly intended to be used to predict what will happen in a dominium scenario and/or in the case of the application of different policy strategies. It can also be used in a prescriptive way, i.e., to define what has to happen to reach a certain objective. It consists of two main elements: transport and land use. Each of them can be decomposed in interacting sub-models, and additionally accounting modules can be attached externally to calculate assessment indicators and pollutant emissions. Furthermore there is the possibility to include external scenarios like demographic transition or growth and changes in car ownership. Its underlying hypothesis is that settlements and activities within them are self-organised and reach an equilibrium state. The different reaction speeds of the two systems (passenger transport system and land use system) are taken into consideration. These different speeds generate a dynamic process which is looking for a dynamic equilibrium, which is disturbed by external transport or land use policy measures, such as road capacity increases, public transport supply changes or land use instruments, e.g. a land value capture tax. Work places are relocated between zones as a function of their accessibility, availability of land, the construction cost and average household income as a proxy for consumption potential and labour cost. Accessibility is calculated as potential to reach work places and shopping facilities with the travel times and travel costs calculated in the transport model. MARS calculates the accessibility for each zone and for each means of transport for each simulation period, and uses this information to redistribute the daily trips of population as well as the relocation of workplaces and housing within the case study area.

Development and availability

MARS is an open source tool, developed at the Institute for Transport Planning and Traffic Engineering at the University of Technology of Vienna and the University of Leeds. MARS development started in 2000 and it has already been implemented in 20 metropolitan areas.

Input/output data and granularity

Input data:

- Number of residents in a zone
- Land rents in model zone
- Area of green land in model zone
- Average household income
- Zone's potential for activity participation (accessibility)
- Abundance of building land
- Cost for building in a zone
- Average household income

Output data:

- Mode/Trip purpose
- Migration sub-model output data:
    - Out /in migration rates
    - Work place out/in migration

Space granularity

NUTS 1 to NUTS 5 and LAU 2 level

Time granularity

- Peak/Off-peak, Day, Year

Geographic scale

- Supra-national, national, supra-municipal, municipal

Objectives

- Scenario testing
- Policy optimisation
- Decision makers training

Technical features

Language and platform requirements:

- MARS model is implemented in a System Dynamics Software environment called VENSIM.
- VENSIM can be run on Windows, Mac OSX v2.4 and higher and also in LINUX by using the WINE emulator.
- MARS input data are stored in Excel.

Graphic interface:

- MARS simulator has a graphic interface which allows the user an easy access to the model.

Detailed information regarding MARS and a free download of the software can be found at http://www.ivv.tuwien.ac.at/forschung/mars-metropolitan-activity-relocation-simulator.html.

5.1.2 UrbanSim

General information

UrbanSim is a highly disaggregated agent-based model designed as an analytical tool to support land-use and transportation planning. The model incorporates the interactions (including constrained lagged interaction) between land use, transportation, the economy and the environment to simulate the development of individual parcels and the decisions of individual households and firms over multiple years (Waddell et al., 2007a).

UrbanSim consists of several sub-models:

- The accessibility model is responsible for maintaining accessibility values for occupants within each traffic analysis zone, including accessibility by residents and employees to shopping and other amenities, to employment, and to the central business district.
- The transition model, which is sub-divided into demographic and economic transition models. The demographic transition model simulates births and deaths in the population of households. Household births are added to a list that will be located later by the location choice model. Household deaths are selected at random and removed from the housing stock, and vacancies are created using an iterative proportional fitting. The economic transition model is responsible for modelling employment creation and loss and is analogous to the demographic transition models.
- The location model, also divided in two sub-models, one for employment and one for households.
- The employment location choice model is responsible for determining a location for each job that has no location. The model is based on a multinomial logit model structure to generate location choice probabilities across a random sampling of location alternatives. Probabilities are used with Monte Carlo sampling to make a determination for each job regarding which of the available locations they will choose. Variables included in the employment location models consider real estate characteristics in the grid cell (price, type of space, density and age), neighbourhood characteristics (average land values, land use mix and employment in each sector) and regional accessibility to population.
- The household location choice model chooses a location for each household that has no current location using a similar approach to the employment location choice model. Variables used in the household location model include attributes of housing in the grid cell (price, density and age), neighbourhood

characteristics (land use mix, density, average property values and local accessibility to retail) and regional accessibility to jobs).

- The mobility model simulates households and firms decisions to move. Decisions on whether to move or not are taken with a probability determined by age and income category of each household and by employment sector applied to each job, for household and firms moves respectively. Movement probabilities are based on historical data. A multinomial logit model is then used to allocate new and moving households to residence locations and jobs to job locations.
- The real estate development model simulates developer choices about place (where to develop) and type (new development and redevelopment of existing structures), using a multinomial logit model. Variables include characteristics of the grid cell (current development, policy constraints and land and improvement values), characteristics of the site location (proximity to highways, arterials, existing development) and regional accessibility to population.
- The land price model simulates land prices of each grid cell as the characteristics of locations change over time, using hedonic regression. A logsum accessibility measure is used, calculated by a travel demand model system.

Development and availability

It is an open source platform developed at the University of Washington by a team led by Dr. Paul Waddell that has been continuously refined and distributed for planning applications around the world for over 15 years. UrbanSim models have been developed in many places worldwide with the operational ones mostly currently used in North America. As UrbanSim brings an important interest amongst researchers, many prototypes have been developed in universities. UrbanSim has been developed for: Detroit (Michigan) (Waddell et al., 2008); Seattle (Washington) (Waddell et al., 2007a); Salt Lake City (Utah) (Waddell et al., 2007b); and San Francisco (California) (Waddell et al., 2007c). UrbanSim has also been applied in Europe with prototypes being implemented for Paris (France) (de Palma et al. 2005), Brussels (Belgium) and Lyon (France) (Patterson et al., 2010) among others. Since December 2012, Synthicity LCC (http://www.synthicity.com) coordinates the development of UrbanSim and provides consultancy services to support its applications. UrbanSim is available for download from the project's website: https://github.com/synthicity/urbansim/wiki

Input/output data and granularity

Inputs:

- Employment data, in the form of geocoded business establishments
- Household data, merged from multiple census sources
- Parcel database, with acreage, land use, housing units, non-residential square footage, year built, land value, improvement value, city and county
- City and County General Plans
- GIS Overlays for environmental features such as wetlands, flood ways, steep slopes, or other sensitive or regulated lands
- Traffic analysis zones
- GIS overlays for any other planning boundaries

- Travel model outputs
- Development Costs

Outputs (by Building, Parcel or Grid cell), generally summarised by zone:

- Households by income, age, size, and presence of children
- Employment by industry and land use type
- Acreage by land use
- Dwelling units by type
- Square feet of non-residential space by type
- Real estate prices

Travel Model Outputs (Zone-to-Zone):

- Travel time by mode by time of day by purpose
- Trips by mode by time of day by purpose
- Composite utility of travel using all modes by purpose
- Generalized costs (time + time equivalent of tolls) by purpose

150 square meter grid cell

Objectives

- Enable stakeholders (planners, public agencies, citizens and advocacy groups) to explore the potential consequences of alternative public policies and investments
- Facilitate more effective democratic deliberation on contentious public actions regarding land use, transportation and the environment, informed by the potential consequences of alternative courses of action that include long-term cumulative effects on the environment, and distributional equity considerations.
- Make it easier for communities to achieve a common vision for the future of the community and its broader environment, and to coordinate their actions to produce outcomes that are consistent with this vision

Technical features

Language and platform requirements:

- The model and user interface is currently compatible with Windows95/NT, Unix, Macintosh, and other platforms supporting Java JDK 1.2; reporting tools are currently implemented in Excel
- The user interface focuses on policy assumptions and the creation and evaluation of scenarios
- The model is implemented using object oriented programming to maximize software flexibility
- The model inputs and results can be displayed using ArcView, Arc/Info, or other GIS software
- Model results are written as ASCII, tab delimited files for external use.

5.1.3 Simulacra

General information

Simulacra is a framework which embraces a series of fast, visually accessible, cross-sectoral static urban models with the objective of testing many different scenarios pertaining to both short and long term urban futures. The models are multisector, dealing with residential, retail/service, and employment location, are highly disaggregate, and subject to constraints on land availability and transport capacities. They have an explicit urban economic focus around transport costs, incomes, and house prices and thus encapsulate simple market-clearing mechanisms. Simulacra is not conformed by single models but acts as a framework in which different variants of a generic model structure can be generated rather quickly. Models in Simulacra are initially static models, simulating more than one sector of the urban system at only one cross-section in time, with the potential for extension to deal with increments of time which are encapsulated within the assumed equilibrium.

The current model deals with four sectors: workplaces defined by employment, residential location defined by population, shopping defined by retail employment and what they define as local industries generated endogenously in the system that we call internal employment. The model links these four activity types through spatial interactions — the journey from work to home defined by trips T linking employment to population, trips S from residential areas to shopping centres, and through implicit industrial linkages measured as accessibilities to employment and to commercial activities defined as A. These activities can be disaggregated in any way and extensions to such classifications are obvious and straightforward.

Trips between home-work and home-retail are simulated using a single constrained interaction model. Internal employment Ml which forms the local industry sector is predicted using a rather different type of model, more akin to those developed by Putnam amongst others. The causal chain from total employment to population to retail and then internal employment is the one first developed by Lowry (1964). The "basic" model can be extended to include urban economy in the flow calculation. This is done by considering flows of people as flows of money. Wages are earned at employment locations and money is spent on travel from work to home and then on housing at the residential end of such a trip. Consumer goods are generated by the population, purchased at the retail end of the trip, and consumed at the residential origin. The residential model is then simulated based on the idea that workers have monies to spend on housing and transport which vary according to the wages they receive at their place of work. This conditions the probability of their journeying to some different location to live, the assumption being that the smaller the difference between their available monies for transport and housing and the cost of travel to that place and the cost of housing there, the greater the probability that they will locate there. This replaces the classic negative exponential travel cost function.

Simulacra includes visually accessible interactive desktop applications. The user can interrogate this model at every stage from the initial stage of data exploration and analysis, through calibration, and thence into evaluating predicted impacts on location and interaction as part of a wider set of 'what-if' style scenarios. This model is based on multiple windows being launched in a systematic way through a toolbar sequence that drives the model input–calibration–prediction processes. Simulacra also includes a web-based model designed following a multitier architecture for enterprise applications which is currently in its development phase. This model is intended to be run on the user's own machine, a dedicated server, or a cluster of computers.

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Development and availability

Simulacra has been developed by CASA-UCL. Development, modifications of the code, new sub-models implementations etc., is only accessible to the group, nevertheless there exist some interfaces that can be accessible to general use. Simulacra has been applied to the Greater London to illustrate how to evaluate scenarios involving changing energy costs (Batty et al., 2013), as well as to examine the impact of two new sites in the Thames Estuary for a new international airport to deal with the congestion at Heathrow and other London airports and to prepare for much more air traffic in the medium-term future, notwithstanding arguments about policies pertaining to a low carbon future.

Objectives

- Rapidly test many different scenarios pertaining to both short-term and long-term urban futures

Technical features

- Most of SIMULACRA interfaces are written in JAVA
- Full information about SIMULACRA can be found at the projects web page http://simulacra.blogs.casa.ucl.ac.uk/

5.1.4 Albatross

General information

Albatross (Learning Based Transportation Oriented Simulation System) is an activity-based model of activity-travel behaviour derived from theories of choice heuristics that consumers apply when making decisions in complex environments. The system forecasts travel demand by simulating individual's decisions related to every facet of activity schedules generally considered relevant for activity-travel analysis. The facets include activity type, duration, start time, trip type, location and transport mode. The system is designed as a rule-based model in which situational, household, institutional and space-time constraints as well as choice heuristics of individuals are explicitly represented in the system. In addition, various situational, temporal, spatial, spatial-temporal and institutional constraints are incorporated. The decision tree is proposed as a formalism to represent an exhaustive set of mutual exclusive rules for each decision step in the model (Arentze and Timmermans, 2000, 2004).

The model is based on a process model, which mimics a priority based scheduling process in which mandatory activities are scheduled first and discretionary activities are scheduled next. Choices made at a particular stage depend on previous choices and in addition on future possibilities. The formalism of decision trees is used to derive the rules that underlie observed activity-travel choices as observed in empirical activity-travel diaries. CHAID-based and F-based tree induction methods are used to derive these decision making rules. Activity-travel choices are simulated probabilistically given a set of spatial-temporal, institutional and household constraints. The system includes 27 decision tables. Thus, the activity-travel behaviour of every member of a fraction of the created population is simulated by activating each of these decision tables sequentially using (i) person, household, institutional and environment information, (ii) simulated decision outcomes of previous scheduling steps and (iii)multiple constraints as input. Because decisions are made probabilistically, processing all individuals requires a substantial amount of computing time. Because the model system is based on the underlying process model, predicted activity-travel patterns emerge from the scheduling process; observed patterns are not a direct input for the estimation of the model system (Rasouli and Timmermans, 2012).

One of the differentiating fractures of the model is that, by assuming that activity participation, allocation and implementation fundamentally take place at the level of the household, the influence of short-term dynamics on long-term dynamics and vice versa is explicitly addressed. At the same time socio-economic and lifestyle variables are considered as relevant parameters for the short/long term decision. According to the authors long-term decisions, such as choice of residence, choice of work and workplace, and purchase of transport modes exert a strong influence on possible activity patterns as the location of the residence and workplace vis-à-vis the transportation system represent the main locations of an activity pattern and are the cornerstones of decisions, while decisions regarding marital status, number of children, and the like, are irreversible or require years to change, and hence have a strong impact on the number and kinds of activities that need to be performed and the constraints that households face also influencing the discretional activities, reflecting an assumed relationship between socio-demographic variables and lifestyle (Arentze and Timmermans, 2004). Authors postulate that the process of program generation depends on the nature of the activities (mandatory versus discretionary), the urgency of completing a particular activity on a specific day as a function of the history of the activity scheduling and implementation process, and the desire to meet particular activity and time-related objectives.

Development and availability

It has been developed for the Dutch Ministry of Transportation, Public Works and Water Management by Arentze and Timmermans (2004) and it is of internal use, not available for general public.

Objectives

- Policy impact analysis

Input data

- Households activity diaries:
    - successive activity
    - information about the nature of the activity
    - activity start and end time
    - activity location
    - transport mode (chain)
    - travel time per mode, if relevant
    - accompanying individuals (alone, other member of household, other)
    - activity planned (yes/no)?
- Data of the physical environment:
    - mode-specific shortest-route
    - travel times

- distances between zones
- size of facilities by sector (measured as total amount of floor space and number of employees per sector)
- opening hours of facilities by sector and day of the week

Output data

The output of the Albatross system is a series of performance indicators, such as distance travelled, emissions, trip-tour ratios, market share of various transport modes, etc.

Space granularity

Activity diaries are considered at a household level while data of the physical environment is aggregated at a zip code level.

5.1.5 Metronamica

General information

METRONAMICA is a generic forecasting tool for planners to simulate and assess the integrated effects of their planning measures on urban and regional development. As an integrated spatial decision support system, it models socioeconomic and physical planning aspects. It simulates the dynamics of changing land use patterns. It consists of a dynamic Cellular Automata spatial land use change model that can optionally include a regional migration model and a transport model for modelling congestion and traffic pressure on the transport network.

METRONAMICA includes 3 possibilities:

- Metronamica SL: containing the land use model as a single layer
- Metronamica ML: containing the land use model and the regional model as multiple layers
- Metronamica LUT: containing the land use model, the regional model and the transport model

The core component METRONAMICA is an explicitly dynamic land use-transportation model applied to the full territory of the area modelled. Land use changes are simulated based on a number of different drivers. First there are external factors such as population growth or the decrease of natural area that determine the demand for different land uses. Population and jobs are divided over the regions, based on how attractive these regions are to people and businesses. This attractiveness depends again on a number of factors such as the existing activity and local characteristics such as the accessibility. Finally, within each region, the land uses for every location are determined based on socio-economic factors (e.g., will a business flourish in this location?), policy options (e.g., are there policy rules in effect that restrict new housing development in this location?) and biophysical factors (e.g., is the soil suited for agriculture here?).

Four building blocks compose METRONAMICA:

- The land-use building block consists of a CA model which has three main characteristics that distinguish it from other CA (Kim, 2011): (i) the strength of the interactions between zones considered follows a distance decay function; (ii) the integration of GIS technology into the model in order to initiate the model, to introduce key extra driving factors such as zoning, suitability, and accessibility, and also to analyse and visualise input data as well as model outcomes; (iii) the total amount of cell transitions is constrained through the use of exogenous variables. In other words, the model does not sum up all possible changes at the local level. The model calculates a ranked score for each cell and then makes an allocation considering the total amount defined.

- The spatial indicators building block develops comparable data bases to derive indicators in order to understand the evolution of urban areas and impacts on the surrounding environment. At the basis of its approach lies the idea that without a spatial approach, any urban indicator set aiming to address sustainability would be incomplete. The spatial indicators model calculates the spatial indicators dynamically with the changing land use on a yearly basis and are available in the form of dynamic maps and numeric outputs.
- The regional building block divides the total jobs in main economic sectors for the whole study area over the regions based on their relative attractiveness and models the levels of activity in different socio-economic sectors that in turn form a restriction on the cell allocation algorithm of the CA model. Specifically, the levels of activity are converted to a number of cells that needs to be allocated to each land use function by the CA model. The level of activity in a sector and region can be expressed in terms of the number of jobs, if dealing with an economic sector, or in terms of the number of people, if dealing with a population sector. This building block is built on a spatial interaction model. The allocation of the growth amongst the regions depends to a large extent on the relative attractiveness of each of the region. In modelling the national socio-economic growth and migration distance also plays a crucial role. The underlying assumption for this is that regions can benefit from other attractive regions, as long as the distance is not too far. Furthermore, people and jobs are reluctant to migrate over greater distances.
- The transport building block simulates transport flows and intensities on the transport network. It is used to simulate the mutual interaction between the transportation system and the land use system. The transport system affects the land use in two ways. The first is via the notion of accessibility and second impact of the transport system is the impact of transport infrastructure and transport intensity on the direct environment. The transport model is based on a classical four step approach. The land use model and regional interaction model serve as input to the transportation model, whereas the transportation model again influences the land use model by means of a local accessibility term and the regional model by means of interregional distances. The transport model is calculated every time step (yearly).

Development and availability

Metronamica was developed by the Dutch-based Research Institute for Knowledge Systems (RIKS) and is a commercial tool.

Input/output data and granularity

METRONAMICA requires the information of two base years to be properly calibrated. This could be any two year as long as there is enough land use change between them. Typically, 10 to 15 year intervals are used, with the final year as recent as possible. This two year data must include:

- GIS (map) data to feed the land use model at the local level (Required for METRONAMICA SL, ML, LUT):
  - Land use:
    - Features: land use classes that are not supposed to change in the simulation, like water bodies or airports
    - Functions: land use classes that are actively modelled, like residential or commercial and in some applications also natural and agricultural land uses
    - Vacant: land use classes only changing as a result of other land use dynamics
  - Suitability:
    - Digital elevation model (DEM) on a grid is required as a base map for the calculation of suitability maps
    - Slope map (can be calculated from the DEM)
    - Aspect map (can be calculated from the DEM)
    - Soil quality map or Geomorphologic map
    - Natural hazards map (fire, flood, landslide, etc.)
    - Pollution maps
  - Zoning: one zoning map for each land use function, covering both the calibration period (between Base year1 and Base year2) and the simulation period (beyond Baseyear2). Zoning maps could be generated with the zoning tool inside of Metronamica, for which the following maps are needed:
    - Actual master plans and zoning plans showing areas that are designated for the development of specific land use categories
    - Maps showing land use restrictions
    - Maps indicating land use policies
  - Accessibility: Networks covering the entire study area should be provided. Separate network maps must be provided for each year in which the network changes
    - Road network
    - Rail network
    - Any other transport systems
    - Key points or nodes
  - Region boundaries map should be provided as a raster map
- Census and other statistical data for the regional model at the regional level (Required for METRONAMICA ML, LUT):
  - Population
  - Employment
- Additional information for the transport model at the regional level (Required for METRONAMICA LUT):
  - Time period
  - Trip purpose
  - Transportation mode
  - Urbanisation class
  - Congestion category

Space granularity

Resolution of the spatial data can be chosen in the range 50 m to 1000 m depending on the precise purpose of the model, the data available and the size of the region modelled. Although technically coarser or finer resolutions are possible, it does not represent the scope of the model and its explorative character.

Objectives

METRONAMICA is a generic forecasting tool for planners to simulate and assess the integrated effects of their planning measures on urban and regional development. As an integrated spatial decision support system, it models socioeconomic and physical planning aspects. It simulates the dynamics of changing land use patterns with the help of a mature model, and has as specific objects:

- Scenario testing
- Policy optimisation
- Decision makers training

Technical features

Computer requirements:

- At least 512 MB of RAM
- A hard disk with at least 1GB free space
- At least 512 MB of RAM
- A hard disk with at least 1GB free space

Graphic interface:

- METRONAMICA is developed in the GEONAMICA software environment. It comes as a stand-alone software application with a user friendly interface. The system includes the MAPCOMPARISON KIT for analysis of model results.
- Both tools use data formats that are compatible with standard GIS packages such as ArcGIS.

5.2 Other land use simulation tools

BOYCE

BOYCE is a combined model of location and travel choice developed by Boyce (Boyce et al.. 1983, 1985; Boyce and Mattsson, 1991; Boyce et al., 1992). This model presents an early example of the synthesis of the residential location model of Wilson (1970) with the network equilibrium approach, thereby making generalised travel costs endogenous to the model (Boyce et al., 1981).

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CUFM (California Urban Futures Model)

CUFM was developed at the University of California, Berkeley. It is a metropolitan simulation model which replicates urban growth patterns and the impacts of development policy at various levels of government. It projects population from the bottom-up, it allocates growth to sites based on development profitability, it embodies the role of accessibility in the development process, and it is operated through the medium of geographic information systems (Landis, 1992, 1993, 1994; Landis and Zhang, 1998a, 1998b).

DELTA

Delta is a land-use/economic modelling package by David Simmonds Consultancy, Cambridge, UK. The overall design consists of four components, namely, the transport model (to which DELTA is linked); the economic model; the urban land-use model; and the migration model. Of these, the transport and urban models work at the level of zones, whilst the migration and economic models work at the broader level of areas. Areas typically correspond to travel-to-work areas, at least within the region of main interest; zones represent finer units within these areas (Simmonds and Still, 1998; Simmonds, 2001; Simmonds and Feldman, 2007).

ILUMASS (Integrated Land-Use Modelling and Transportation System Simulation)

ILUMASS embeds a microscopic dynamic simulation model of urban traffic flows into a comprehensive model incorporating changes of land use, the resulting changes in transport demand, and the impacts of transport on the environment. Microsimulation modules include models of demographic development, household formation, firm lifecycles, residential and non-residential construction, labour mobility in the regional labour market and household mobility in the regional housing market. These modules are closely linked with the models of daily activity patterns and travel and goods movements of the transport part (Strauch et al., 2005).

ILUTE (Integrated Land Use, Transportation, Environment)

The modeling system developed by several Canadian universities. It is an integrated urban modeling system that consists of a behavioural core of the four inter-related components, namely, Land Development, Location Choice, Activity/Travel, Auto Ownership. Each of these behavioural components involves a complex set of sub-models that incorporate supply/demand interactions and interact with each other. For example, land use evolves in response to location needs of households and firms, and people relocate their homes and/or jobs at least partially in response to accessibility factors (Doherty and Miller, 2000; Miller and Salvini, 2001).

IMREL (Integrated Model of Residential and Employment Location)

It was developed at the Royal Institute of Technology in Stockholm by Anderstig and Mattsson (1991, 1998). It is based on random utility theory with fixed travel costs. The IMREL model consists of two sub-models: a sub-model for residential location choice (RES) and a sub-model for employment location choice (EMP). It is a static allocation model requiring an external transport model. It represents policy by placing limits on the number of workplaces or households that can locate in a zone. IMREL produces a pattern of land use based on one set of accessibilities given by a particular run of the transport model. Therefore it is cross-sectional rather than incremental. The residential and employment sub-models are integrated. The location of households responds to the employment pattern and the employment choice responds to the pattern of households. The model iterates until equilibrium is obtained (DSC & MEP, 1999; Pfaffenbichler, 2003)

IRPUD

The model of the Dortmund region developed at the University of Dortmund. The IRPUD model is a simulation model of intraregional location and mobility decisions in a metropolitan area (Wegener, 1982; 1983; 1985; 1994; Wegener and Spiekermann, 1996). It receives its spatial dimension by the subdivision of the study area into zones connected with each other by transport networks containing the most important links of the public transport and road networks coded as an integrated, multimodal network including all past and future network changes. It receives its temporal dimension by the subdivision of time into periods of one or more years (Wegener, 1998).

ITLUP (Integrated Transportation and Land Use Package)

The ITLUP model (Putman, 1983) combines two separate components: a land use model and a transportation network model. In the original version each component was a modification of an existing model. The land-use component was based on a modified Garin-Lowry model. The network model was a conventional capacity-restrained incremental-assignment model. Later the land-use component was again revised and named Disaggregated Residential Allocation Model (DRAM) (Pfaffenbichler, 2003; Putman, 1991, 1998).

KIM

The non-linear urban equilibrium model developed at the University of Illinois at Urbana-Champaign is an integrated urban simulation model with a non-linear structure, combining both commodity-flow model and urban activity model. Using partial linearisation techniques with a numerical solution method, this model was extended for three-dimensional urban activity (Kim, 1989; Rho and Kim, 1989).

LILT (Leeds Integrated Land-Use/Transport)

The LILT model represents the relationships between transport supply (or costs) and the spatial distribution of population housing, employment, jobs, shopping and land utilisation (Mackett, 1990a). LILT is an allocation model which locates net change in population, housing and jobs from exogenous forecasts. It links the trip distribution and mode split stage of a four stage transport model with a Lowry type land use model (Mackett, 1983, 1990b, 1991a, 1991b).

MEPLAN

The integrated modelling package MEPLAN, developed by Marcial Echenique & Partners, is a mathematical framework and software package for modeling the spatial economies of cities or regions. It contains a fairly detailed and comprehensive transportation planning model and the influence of transportation conditions on the location of various land uses are core to the function and purpose of the model (Abraham, 1998; Echenique et al., 1969; Echenique and Williams, 1980; Echenique, 1985; Echenique et al., 1990; Hunt and Echenique, 1993; Hunt and Simmonds, 1993; Hunt 1994).

METROSIM

The microeconomic land-use and transport model METROSIM, developed for the New York Metropolitan Area, is the outcome of many years of theoretical research and practical model building by Alex Anas (DSC & MEP, 1999). METROSIM is designed to forecast the interdependence effects of transport and land use at the metropolitan level for US Metropolitan Planning Organizations. The structure is based on a series of economic relationships. METROSIM can be used as a static model to produce a long-run equilibrium forecast for location and travel patterns. Alternatively, it might be operated as a quasi-dynamic model operating in yearly increments producing yearly changes (Anas, 1992, 1994, 1998).

MUSSA

This '5-Stage Land-Use Transport Model' developed for Santiago de Chile has been primarily used as a research tool. MUSSA is linked to a conventional large scale, four stage transport model. MUSSA predicts the total (cross-sectional) land-use pattern given a particular set of accessibility inputs. MUSSA and the linked transport model are iterated until equilibrium (at least to some equilibrium conditions, if not to complete equilibrium). The accessibility measures are designed to measure consumer surplus of transport users. This allows transport benefits to be measured in the land-use model. The land-use model itself is disaggregated. The probabilities of location of all households and firms in a sample derived from surveys are calculated. As in conventional disaggregate transport modeling, future growth is represented by changing the expansion factors attached to each record in the sample. The location process involves a sophisticated representation of households'/firms' willingness to pay for different locations and of landlords' preference for taking the highest bid for each available dwelling/site. MUSSA is based on several equilibrium assumptions. To use it in dynamic modelling the number of equilibrium assumptions needs to be reduced (Martínez, 1992; Martínez and Donoso, 1996).

PECAS (Production, Exchange and Consumption Allocation System)

Developed at the University of Calgary, PECAS is an urban and regional modeling tool to support transportation and economic planning. It contains two principal models: Activity Allocation (AA): an aggregate, equilibrium Spatial Input-Output Model; Spatial Development (SD): a disaggregate State-Transition model. It was developed initially as part of an Oregon Department of Transportation (ODOT) Statewide Modeling project as a replacement for a 1st generation statewide model using TRANUS. Recently, CalTrans implemented a contract with UC Davis to support development of a California Statewide PECAS model, and to support MPOs within the state in the development of metropolitan level PECAS models (Parsons Brinckerhoff Ohio and Hunt Analytics, 1999; Hunt and Abraham, 2003; Waddell, 2011).

POLIS (Projective Optimization Land Use Information System)

POLIS was developed by Prastacos for the Association of Bay Area Governments. The allocation process in POLIS is based on several criteria, some reflecting the behavior of individuals and some describing physical and planning constraints: travel-to-work and shopping behavior; the availability and attractiveness of housing; and the current levels of nearby employment determine residential choice. Retail activity is located in proximity to population centres to maximise sales revenue. The locational patterns of the other industries are influenced by the accessibility to labour supply, the proximity to other similar industries and local development policies. POLIS is a structured mathematical programming, optimisation problem, that is, the allocation of population and employment is optimised with respect to an objective function or goal while at the same time satisfying planning constraints. POLIS converges after several iterations on a solution that optimally allocates jobs and households, subject to the constraints. It results in housing, employment and trip flow patterns which are consistent with each other and the land use constraints (Prastacos, 1986; Caindec and Prastacos, 1995).

RURBAN (Random-Utility URBAN)

RUBAN model was developed by Miyamoto (Miyamoto et al., 1986). RURBAN was initially intended for application on small size zones. This is because land use structures in Japan or Asian countries are so complex that large size of zones cannot well describe the urban structure. First, in order to segment the demand side in the land market, the urban locators are classified according to their characteristics, i.e., a limited number of locator groups are defined. These groups represent discrete options in the random rent-bidding analysis. The supply side of the land market is segmented by aggregating individual sites into zones based on their locational conditions. The zones are regarded as discrete options in the location choice analysis with random utility. The land market is grasped from two viewpoints of locators and sites. If a locator chooses a particular site, it implies that the site gives the locator the highest utility compared with the alternative sites. On the other hand, the locator must bid the highest rent among the alternative locators for the site (Miyamoto and Kitazume, 1989; Miyamoto and Udomsri, 1996; Miyamoto et al., 2007).

SLEUTH model (Slope, Land Use, Exclusion, Urban Extent, Transportation, Hillshade)

The SLEUTH model is a CA model, developed with predefined growth rules applied spatially to gridded maps of the cities in a set of nested loops, which was designed to be both scalable and universally applicable (Silva and Clarke, 2002). SLEUTH is composed of four growth rules to simulate urban growth: (i) spontaneous growth rule is responsible for urbanisation of randomly selected pixels; (ii) the urban pixels created by the spontaneous growth are then tested for the probability to become new urban spreading centres under the new spreading centre growth rule; (iii) the edge growth rule is responsible to stem the organic growth of new urban spreading centres; and finally (iv) the road influenced growth rules determines the urban growth along the transportation network. SLEUTH also has a functionality termed ‘self-modification’ (Clarke et al., 1997) which allows the growth coefficients to change throughout the course of a model run and which is intended to simulate more realistically the different rates of growth that occur in an urban system over time (Jantz et al., 2004; KantaKumar et al., 2010).

STASA

STASA was developed for the metropolitan region of Stuttgart. The modeling of the spatio-temporal patterns of a system consisting of different sub-models (population, transport, production, etc.) is used to demonstrate the necessity to link different theoretical frameworks. External shocks may sometimes require a modification of the system under consideration in the sense that new dynamic variables appear or previously useful variables disappear (Haag, 1990; Haag and Binder, 2008).

TELUM (Transportation Economic and Land Use Model)

TELUM uses the ITLUP (Putman, 1983, 1991, 1998) equations to predict the location and growth of residential and non-residential development for up to thirty years. Predictions are based on the analysis of current year and a lag year residential and non-residential development, the locations of transportation improvements, and changes in land use conditions and inter-zonal travel cost over time. TELUM consists of three sub-models, namely TELUM-EMP, TELUM-RES and LANCON. TELUM-EMP forecasts the future distribution of employment and was developed based on the EMPAL model of ITLUP. TELUM-RES, developed based on the DRAM model of ITLUP, predicts the distribution of households and population in each forecast year. LANCON computes the amount of land consumed in each zone based on the assigned residential and employment distribution and the developable (supply) land in that zone (Duthie et al., 2007; Kockelman et al., 2008; Putman, 2007).

TLUMIP

TLUMIP is the land-use transport model of the US State of Oregon developed in the Oregon Transport and Land Use Model Integration Program (ODOT, 2002). The primary goal of TLUMIP was developing an integrated transportation, land use and economic model for use in transportation planning and policy analyses at the regional and state-wide levels. The TLUMIP models simulate land use and travel behavior relying on various data, from business sector exports to transportation operator characteristics. The models are a valuable complement to more traditional urban and MPO travel demand models. The first generation of the model was completed in 1999, the second generation version in 2007. The model is known as the Statewide Integrated Model (SWIM2). It has been used successfully to evaluate large-scale, complex policy issues (ODOT, 2003).

TRANUS

TRANUS is a transport and land-use model developed by de la Barra (1982). TRANUS is a software package similar to MEPLAN (Hunt and Simmonds, 1993). The TRANUS model is based on a hierarchical structure. TRANUS has a highly integrated architecture and relies on the random utility approach. It simulates the location of activities in space, land use, the real estate market and the transportation system. It may be applied to urban or regional scales. It is specially designed for the simulation of the probable effects of projects and policies of different kinds in cities and regions, and to evaluate the effects from economic, financial and environmental points of view. TRANUS is developed and maintained by Modelistica, which also supplies support and consulting services (de la Barra et al. 1984; de la Barra 1989, 1998; MOD, 2007).

TRESIS (Transportation and Environment Strategy Impact Simulator)

TRESIS was developed at the University of Sydney. It is a decision support system intended to assist planners to predict the impact of transport strategies and to make recommendations based on those predictions. A key focus of the simulator is the richness of policy instruments such as new public transport, new toll roads, congestion pricing, gas guzzler taxes, changing residential densities, introducing designated bus lanes,

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implementing fare changes, altering parking policy, introducing more flexible work practices, and the introduction of more fuel efficient vehicles. The appropriateness of mixtures of policy instruments is gauged in terms of a series of performance indicators such as impacts on greenhouse gas emissions, accessibility, equity, air quality and household consumer surplus (Ton and Hensher, 2001).

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Annex I. Abbreviations and Acronyms

ABM	Agent Based Modelling
CA	Cellular Automata
CBD	Central Business District
GIS	Geographic Information System
ICT	Information and Communication Technologies
LAU	Local Administrative Unit
LSCP	Location Set Covering Problem
MCLP	Maximal Covering Location Problem
MODL	Multi Objective Dynamic Location
NUTS	Nomenclature of Territorial Units for Statistics
LAU	Local Administrative Unit
PMP	P-Median Problem
PCP	P-Center Problem